EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 2005/2006

March/April' 2008

SECOND SEMESTER

MT 218 - FIELD THEORY

Proper & Repeat

Answer all questions Time:Two hours

Q1. (a) State the Gauss's theorem in Electric field.

(b) A spherical conductor of radius a carrying a charge e_1 is surrounded by a concentric spherical conducting sheet of radius b and carrying a charge e_2, both conductors being insulated. Find the potential at a point between the spheres.

If the inner conductor is connected by a fine insulated conducting wire, passing through a small hole in the outer conductor, to a distant uncharged, insulated spherical conductor of radius c, prove that the latter will be raised to a potential $\dfrac{e_1 b + e_2 a}{4\pi\epsilon_0 b(a+c)}$.

(c) Show that the potential (ϕ) at a point distance r_1 and r_2 respectively from centers of a long parallel pair of wires of negligible cross section and having equal and opposite linear charge densities λ coulomb per meter is given by $\phi = \dfrac{\lambda}{2\pi\epsilon_0}\log\left(\dfrac{r_2}{r_1}\right)$.

Q2. (a) State the poisson's equation in electric field.

(b) The expressions for electric scalar potential associated with volume charge distributions are given below. Determine the electric field intensity E in each case by performing gradient operation. Then by taking the divergence of the field, evaluate the volume charge distribution ρ at the origin.

i. $V = 2(x+1)^2(y-1)^2z^2$ (Cartesian System);

ii. $V = R^2\cos^2\theta$ (Spherical System).

Where $\nabla V = \frac{V}{R}\hat{R} + \frac{1}{R}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$ (Spherical System).

(c) Two metallic spheres mounted side by side with the spacing of d mm between their edges are connected across a high voltage transmission line. To preclude the possibility of voltage surges exceeding 20 kV on the line, what spacing d should be used, if the spheres are 100 mm in diameter? Breakdown strength of air 3.0 kV per mm.

Q3. (a) Using Ampere's circuit law and Biot-Savart law, prove that $\nabla^2\phi = 0$, where ϕ is scalar potential.

(b) Show that the equivalence between Biot-Savart and Ampere's laws will be brought out by determining the magnetic field $\overrightarrow{B}$ due to an infinitely long conductor carrying a steady current through it.

(c) A long thin flat strip of metal is of width W and has a current I flowing along it. Find the magnetic induction B at a point P in the plane of the strip at a distance b from the nearest edge.

Q4. (a) State the Gauss's theorem in gravitational field.

(b) State and prove the Kepler's third law.

Where the equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(c) The density of a sphere of mass M and radius a, at a distance r from the center of the sphere is given by $\rho = \rho_0\left(\frac{a-r}{a}\right)$, where ρ_0 is constant.

Show that the gravitational attraction at point P at a distance $x(< a)$ from the center of the sphere is ,

$$GMx\left(\frac{4a-3x}{a^4}\right)$$

. Show also that the potential at P is $-\frac{GM}{a^4}(2a^3 - 2ax^2 + x^3)$ and show that the gravitational potential energy is $\frac{26GM^2}{35a}$.