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EASTERN UNIVERSITY, SRI LANKA
THIRD EXAMINATION IN SCIENCE 2005/2006
August/September' 2007
FIRST SEMESTER
MT 304 - GENERAL TOPOLOGY
Proper & Repeat

Answer all questions | Time:Two hours

Q1. (a) Define the following terms:

i. Topology on a set;

ii. Closure of a set. [20 marks]

(b) Let X be a non-empty infinite set and let τ be the family consisting of ϕ and all subsets of X whose complements are finite. Prove that τ is a topology on X. [30 marks]

(c) Let (X, τ) be a topological space and let $A, B, C \subseteq X$. Define $Fr(M) = \overline{M} \backslash M^o$ for any subset M of X. Prove that

i. If B is closed then $Fr(B) \subseteq B$.

ii. The set B is both open and closed if and only if $Fr(B) = \phi$.

iii. $\bar{A} = Fr(A) \cup A$. [35 marks]

(d) Is the union of two topologies on a set X again a topology? Justify your answer. [15 marks]

Q2. (a) Define the following in a topological space(X, τ):

i. Base;

ii. Disconnected set.

[20

(b) Let $\mathbb{B}$ be a class of subsets of a non-empty set X. Prove that $\mathbb{B}$ is a b
some topology τ on X if and only if it satisfies the following properties

i. $X = \bigcup_{i \in I} B_i, \quad B_i \in \mathbb{B};$

ii. For any $B_\alpha, B_\beta \in \mathbb{B}$, $B_\alpha \bigcap B_\beta = \bigcup_{i \in I} B_i$, that is,

$B_\alpha \bigcap B_\beta$ is the union of members of $\mathbb{B}$.

[35 m

(c) Prove that a topological space (X, τ) is disconnected if and only if there ex
non empty proper subset of X, which is both open and closed. [20 m

(d) Let (X, τ_X) and (Y, τ_Y) be two topological spaces and let $f : X \to Y$
continuous function. Prove that if $A \subseteq X$ is connected, then the image of
connected.

[25 ma

Q3. (a) Define the following in a topological space (X, τ):

i. Compact set;

ii. Sequentially compact set.

[20 ma

(b) Let (X, τ) be a topological space and let (Y, τ_Y) be its subspace. Prove th
is compact in (Y, τ_Y) if and only if A is compact in (X, τ). [25 ma

(c) Prove that continuous image of a compact set is compact. [15 ma

(d) Let A, B be two compact sets in a topological space (X, τ). Show that $A \cup$
compact.

[20 ma

(e) Is $A = (0, 1)$ on the real line $\mathbb{R}$ with usual topology compact? Justify your
swer.

[20 ma

Q4. (a) Define the Frechet space and the Hausdorff space. [15 ma

(b) Let (X, τ_1) and (Y, τ_2) be two topological spaces and let $f : X \to Y$. Show
f is continuous on $A \subseteq X$ if and only if $f(\overline{A}) \subseteq \overline{f(A)}$. [30 ma

(c) Prove that every Hausdorff space is Frechet space. Is the converse true?
Justify your answer.

[25 ma

(d) Prove that a topological space is a Frechet space if only if every singleton subset of X is closed.

[30 marks]