EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE (2004/2005)

SECOND SEMESTER (Aug./Sep.'2007)

Special Repeat

MT 307 - CLASSICAL MECHANICS III

Answer all questions

Time: Three hours

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1. Two frames of reference S and S' have a common origin O and S' rotates with an angular velocity $\underline{\omega}$ relative to S. If a moving particle P has its position vector as $\underline{r}$ relative to O at time t show, by carefully defining the derivative, that

   (a) $\dfrac{d\underline{r}}{dt} = \dfrac{\partial \underline{r}}{\partial t} + \underline{\omega} \wedge \underline{r}$ and

   (b) $\dfrac{d^2\underline{r}}{dt^2} = \dfrac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \wedge \dfrac{\partial \underline{r}}{\partial t} + \dfrac{\partial \underline{\omega}}{\partial t} \wedge \underline{r} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r})$.

   A projectile located at a point O co-latitude λ is fired with velocity v_0 in a southward direction making an angle α with the horizontal.

   (a) Find the position of the projectile after time t,

   (b) Prove that after time t the projectile is deflected towards the east from its original vertical plane of motion by the amount

$$\frac{1}{3}\omega g \sin \lambda t^3 - \omega v_0 \cos(\alpha - \lambda) t^2,$$

(c) Prove that when the projectile returns to a point on the horizontal plane through O, at a distance

$$\frac{4\omega v_0^3 \sin^2 \alpha}{3g^2}\{3\cos\alpha\cos\lambda + \sin\alpha\sin\lambda\}$$

to the west of the point of projection.

2. With usual notations, obtain the equations of motion for a system of N particles in the following forms;

(a) $\dfrac{d\underline{H_G}}{dt} = \displaystyle\sum_{i=1}^{N} \underline{R_i} \wedge \underline{F_i},$

(b) $\underline{\Gamma}_A = \dfrac{d\underline{H_G}}{dt} + (\underline{r}_G - \underline{r}_A) \wedge M\underline{f}_G,$

where

$$\underline{\Gamma}_A = \sum_{i=1}^{N} (\underline{r}_G - \underline{r}_A) \wedge \underline{F}_i$$

is the moment of the external forces acting on the system about on the system about a moving point A in the system.

A solid of mass M is in the form of a tetrahedron $OXYZ$. The edges OX, OY and OZ of which are mutually perpendicular rests with XOY on a fixed smooth horizontal plane and YOZ leaning against a smooth vertical wall. The unit normal to the rough face XYZ is $\underline{n}$. A heavy uniform sphere of mass m and center C rolls down the face XYZ causing the tetrahedron to acquire a velocity $V\underline{j}$ where $\underline{j}$ is the unit vector along OY. If $\overrightarrow{OC} = \underline{r}$, then prove that

$$(M+m)V - m\dot{\underline{r}} \cdot \underline{j} = constant,$$

and that

$$\frac{7}{5}\ddot{\underline{r}} = \underline{f} - \underline{n} \cdot (\underline{n} \cdot \underline{f}),$$

where $\underline{f} = \underline{g} + \dot{V}\underline{j}$ and $\underline{g}$ is the acceleration of gravity.

3. (a) With usual notations, show that, for the motion of a top with its tip on a perfectly rough horizontal floor,

$$s + \dot{\phi}\cos\theta = constant = n,$$

$$A\dot{\phi}\sin^2\theta + Cn\cos\theta = constant = k,$$

$$A(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta) + 2mgh\cos\theta = constant,$$

where s is the spin angular velocity of the top.

(b) Let $u = \cos\theta$. Prove that

i. $\dot{U}^2 = (\alpha - \beta u)(1 - u^2) - (\gamma - \delta u)^2 = f(u)$ (say)

where $\alpha = \dfrac{2E - Cn^2}{A}$, $\beta = \dfrac{2mgh}{A}$, $\gamma = \dfrac{k}{A}$ and $\delta = \dfrac{Cn}{A}$.

ii. $t = \int \dfrac{du}{\sqrt{f(u)}} + constant.$

4. With the usual notations, obtain Euler's equations of motion for a rigid body having a point fixed, in the following form:

$$A\dot{\omega}_1 - (B - C)\omega_2\omega_3 = M_1,$$
$$B\dot{\omega}_2 - (C - A)\omega_1\omega_3 = M_2,$$
$$C\dot{\omega}_3 - (A - B)\omega_1\omega_2 = M_3.$$

A solid consist of two equal uniform right circular cones each having height b and vertex angle right angle, rigidly joined at the vertex O, such that their axis in the same straight line.

If O is fixed and the solid is set to rotate about a common generator of cones with angular velocity $\underline{\omega}$ under no forces except gravity and the reaction at O. Show that the solid will be rotate about the frame generator after a time $\dfrac{10\Pi\sqrt{2}}{3\omega}$.

Given that the principal moment of inertia $A = B = \dfrac{3}{4}Mb^2$ and $C = \dfrac{3}{10}Mb^2$.

5. Deduce Lagrange's equations for impulsive motion from Lagrange's equations for a holonomic system of the form

$$\left(\frac{\partial T}{\partial \dot{q}_j}\right)_2 - \left(\frac{\partial T}{\partial \dot{q}_j}\right)_1 = S_j, \qquad j = 1, 2, \cdots,$$

where subscript 1 and 2 denote quantities before and after application of the impulsive motion.

A uniform rod AB of length $l$and mass m is at rest on a horizontal smooth table. An impulse of magnitude I is applied to one end A of the perpendicular to it. Prove that, immediately after the application of impulse.

(a) the one end A of the rod AB has the velocity of magnitude $\frac{4I}{m}$,

(b) center of mass of the rod AB has the velocity of magnitude $\frac{I}{m}$,

(c) the rod rotates about the center of mass with angular velocity of magnitude $\frac{6I}{m}$.

6. (a) Define **Poission bracket**.

Show that, for any function $F(p_j, q_j, t)$,

$$\dot{F} = [F, H] + \frac{\partial F}{\partial t}$$

where H is a Hamiltonian.

(b) With the usual notations, prove that:

i. $\frac{\partial}{\partial t}[f, g] = \left[\frac{\partial f}{\partial t}, g\right] + \left[f, \frac{\partial g}{\partial t}\right]$,

ii. $[f, q_k] = -\frac{\partial f}{\partial p_k}$,

iii. $[f, p_k] = \frac{\partial f}{\partial q_k}$.

(c) Show that, if f and g are constants of motion then their Poission bracket $[f, g]$ is a constant of motion.