# EASTERN UNIVERSITY, SRI LANKA

## DEPARTMENT OF MATHEMATICS

### FIRST EXAMINATION IN SCIENCE - 2009/2010

### FIRST SEMESTER (June/July, 2011)

### AM106 - TENSOR ANALYSIS

Answer all questions                                    Time : One hour

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1. (a) Write the transformation equation for the following tensors:

    i. $A^{ms}_{qr}$;

    ii. $B^{pqr}_{mn}$;

    iii. $C_{ijk}$.

   (b) Define the terms symmetric and skew-symmetric tensors.

    i. If $ds^2 = g_{ij}\, dx^i\, dx^j$ is an invariant, then show that $g_{ij}$ is a symmetric covariant tensor of rank two.

    ii. If $A^{pq}$ and $B_{rs}$ are skew-symmetric tensors, then show that $C^{pq}_{rs} = A^{pq} B_{rs}$ is a symmetric tensor.

   (c) The covariant components of a tensor in rectangular co-ordinate system are $yz$, $3$, $2x+y$. Find its covariant components in cylindrical co-ordinates $(\rho,\ \phi,\ z)$.

2. (a) Define the Christoffel's symbols of the first and second kind.

(b) Determine the Christoffel's symbols of the second kind for the line element given by

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta \, d\phi^2.$$

(c) With the usual notations, prove the following:

i. $\dfrac{\partial g_{pq}}{\partial x^m} = [pm, q] + [qm, p]$;

ii. $\dfrac{\partial g^{pq}}{\partial x^m} = -g^{pn}\Gamma^q_{mn} - g^{qn}\Gamma^p_{mn}$;

iii. $\dfrac{1}{2g}\dfrac{\partial g}{\partial x^m} = \Gamma^j_{jm}.$