



**EASTERN UNIVERSITY, SRI LANKA**  
**DEPARTMENT OF MATHEMATICS**  
**FIRST EXAMINATION IN SCIENCE -2009/2010**  
**FIRST SEMESTER (June/July, 2011)**  
**AM 151 - MATHEMATICA**  
**(PROPER)**

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Answer all Questions

Time: Two hours

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Attention: You are not allowed to access the facility of Mathematica Help option.

And you may use **Basic Input Palatte** for entering symbols only.

1. Use the Mathematica command(s) to solve the following problems.

(a) Find the integer closest to  $\sqrt{159}$ .

(b) Compute  $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$ .

(c) Compute a numerical approximation of  $\left(1 + \frac{1}{2}\right) \left(1 + \frac{1}{2} + \frac{1}{3}\right) \dots \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}\right)$ .

(d) Determine whether the expression  $1 + x \sin y + x^2 \cos y + x^5 e^y$  is a polynomial in  $x$ . Is it a polynomial in  $y$ ?

(e) Find the partial fraction expansion of  $\frac{(x-1)^6}{(x^2+1)(x+1)^2(x-4)}$ .

(f) Plot the graph of  $y = \sin x$  from 0 to  $\pi$ .

(g) Sort the letters of the word "MISSISSIPPI" alphabetically.

(h) Construct a  $5 \times 5$  matrix having the first five primes as diagonal entries and zeros elsewhere.

(i) Create a list contains all the subsets of  $\{a, b, c, d, e\}$  which contain precisely three elements. How many are there?

- (j) The 20<sup>th</sup> prime is 71. Find all the numbers less than 71 which are not prime.
2. (a) The binomial coefficient  $C(n, k) = \frac{n!}{k!(n-k)!}$  can be expressed as

$$\binom{n}{k} = \binom{n-1}{k-1} \binom{n-2}{k-2} \cdots \binom{n-k+1}{1}$$

for more efficient computation. Use this representation to compute  $C(10, 4)$ .

- (b) The area enclosed by a triangle whose sides have length  $a$ ,  $b$  and  $c$  is given by Heron's formula

$$K = \sqrt{s(s-a)(s-b)(s-c)},$$

where  $s = \frac{a+b+c}{2}$ . Express the area of a triangle as a function of  $a$ ,  $b$  and  $c$  and compute the area of the triangle whose sides are 3, 4 and 5.

- (c) If  $p$  dollars is invested for  $t$  years in a bank account paying an annual interest rate of  $r$  compounded  $n$  times per year, the amount of money after  $k$  periods is

$$p \left(1 + \frac{r}{n}\right)^k$$

dollars. If 1000 dollars is invested in an account paying interest rate 6 compounded quarterly, make a table with suitable headings showing how much money is accumulated during a three year period.

- (d) If  $x$  is an approximation to  $\sqrt{a}$ , then it can be shown that  $\frac{1}{2} \left(x + \frac{a}{x}\right)$  is a better approximation. Use the command **NestList** to observe the first 10 approximation obtained in computing  $\sqrt{3}$ , starting with  $x = 100$ .

- (e) The logistic equation for population growth given by  $\frac{dp}{dt} = ap - bp^2$ , was discovered in the mid-nineteenth century by the biologist Pierre Verhulst. The constant  $b$  is generally small in comparison to  $a$  so that for small population size  $p$  the quadratic term in  $p$  will be negligible and the population will grow approximately exponentially. For large  $p$ , however, the quadratic term serves to slow down the rate of growth of the population. Solve the logistic equation and sketch the solution for  $a = 2$ ,  $b = 0.005$ , and an initial population  $p_0 = 1$ (thousand). Then determine the limiting value of the population as  $t \rightarrow \infty$ .