PER ARDUA AD SCIENTIAM

EASTERN UNIVERSITY, SRI LANKA
THIRD EXAMINATION IN SCIENCE - 2005/2006
FIRST SEMESTER (August/September,2007)
MT 310 - FLUID MECHANICS
(SPECIAL REPEAT)

Answer all Questions **Time: Two hours**

Q1. (a) With the usual notations, derive the equation of continuity of a fluid flow in the form of

$$\frac{d\rho}{dt} + \rho \vec{\nabla} \cdot \vec{q} = 0,$$

where $\frac{d}{dt}$ denotes the differentiation following the fluid particle.

(b) For an incompressible fluid, show that

$$u = \frac{kx}{(x^2+y^2+z^2)^{\frac{3}{2}}}\,\hat{i}, \quad v = \frac{ky}{(x^2+y^2+z^2)^{\frac{3}{2}}}\,\hat{j}, \quad w = \frac{kz}{(x^2+y^2+z^2)^{\frac{3}{2}}}\,\hat{k}$$

are the velocity components of a possible fluid motion.

(c) Determine the condition that

$$u = ax + by, \quad v = cx + dy$$

would be the velocity components of an incompressible fluid. Show that the streamlines of this motion are rectangular hyperbolas when the motion is irrotational.

Q2. (a) With the usual notations, write the Euler's equation of motion of a fluid and use it to show that an incompressible, irrotational steady flow is given by

$$\int \frac{p}{\rho} + \frac{1}{2}\,q^2 + \Omega = \text{constant}.$$

(b) A long pipe of length l has slowly tapering cross-section. It is inclined at an angle α to the horizontal and water flows steadily through it from the upper end to lower end. The section at the upper end has twice the radius of the lower end. At the lower end the water is delivered at atmospheric pressure π. If the pressure at the upper end is twice atmospheric. Show that the velocity of the delivery is

$$\left\{\frac{32}{15}\left(gl\sin\alpha + \frac{\pi}{\rho}\right)\right\}^{\frac{1}{2}},$$

where ρ is the density of the water.

Q3. A source S and a sink T of equal strengths m are situated within the space bounded by a circle whose center is the origin O. If S and T are at equal distances from O on opposite sides of it and on the same diameter AOB. Show that the velocity of the liquid at any point P is

$$2m\left(\frac{OS^2 + OA^2}{OS}\right)\left(\frac{PA.\,PB}{PS.PS'.PT.PT'}\right),$$

where S' and T' are the inverse of S and T with respect to the circle.

Q4. A spherical cavity of radius R_0 containing gas at pressure p_0 begins to expand rapidly in surrounding unbounded liquid. Let R be the radius of the cavity at time t, p_1 the pressure of the gas which is assumed to expand according to the law

$$\frac{p_1}{p_2} = \left(\frac{R_0^3}{R^3}\right)^{\gamma}.$$

Show that the pressure p of the liquid at a point distance r from the center of the cavity is given by

$$\frac{p}{\rho} + \frac{1}{2}\left(\frac{R^2\dot{R}}{r^2}\right)^2 - \frac{R^2\ddot{R} + 2R\dot{R}^2}{r} = f(t),$$

where $\dot{R} = \dfrac{dR}{dt}$, gravity being neglected. If $\gamma = \dfrac{4}{3}$ and $R = (1+n)R_o$, show that the solution of the above equation is given by

$$\frac{ct}{R_o} = \left(1 + \frac{2n}{3} + \frac{n^2}{5}\right)\sqrt{2n},$$

where $c^2 = \dfrac{p_0}{\rho}$.