EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE (2007/2008)

FIRST SEMESTER(December/January, 2008)

MT 304 - GENERAL TOPOLOGY

(PROPER & REPEAT)

Answer all Questions — Time: Two hours

Q1. (a) Define the following:

i. frontier point,

ii. exterior point.

(b) Prove that the intersection of two topologies for a set X is again a topology.

(c) Prove that $A \cup A'$ is a closed set, for any set A where A' is the derived set of A.

(d) Let $(\mathbb{R}, \tau)$ be a topological space induced by the absolute value metric. Let $A = (0, 1]$. Find $Fr(A)$, the frontier of the set A.

(e) Let A be a subset of a topological space(X, τ).Show that:

i. $(\overline{A})^c = (A^c)^o$;

ii. $\overline{A^c} = (A^o)^c$.

Q2. (a) Define the following terms:

i. subspace of a topological space (X, τ);

ii. connected set.

(b) Let (X, τ_1) be a subspace of a topological space (Y, τ_2) and let (Y, τ_2) b subspace of a topological space (Z, τ_3). Show that (X, τ_1) is a subspac (Z, τ_3).

(c) Prove that the continuous image of a connected set is connected.

(d) Let (X, τ) be a topological space. Prove that if X is connected, then X can be expressed as the union of two disjoint non empty closed sets.

Q3. Prove or disprove each of the following statements:

(a) In the usual topology on $\mathbb{R}$, the subset $(0, 1)$ is compact.

(b) (X, τ) is a compact topological space if and only if the collection $\{A_\alpha \mid \alpha \in I, \ \ A_\alpha \text{ are closed sets}\}$ satisfies the finite intersection property, t is $\bigcap_{\alpha \subset I} A_\alpha \neq \emptyset$.

(c) In any topological space (X, τ) and $A \subseteq X$ a set G is open in A if and onl G is open in X.

(d) If (X, τ) is a topological space, then every finite subset A of X is compact.

Q4. Define the *Frechet space* and the *Hausdorff space.*

(a) Prove that every Hausdorff space is Frechet space. Is the converse true? Jus your answer.

(b) Prove that a topological space X is a Frechet space if and only if every single subset of X is closed.

(c) If A is a non empty proper compact subset of a Hausdorff space, prove tha is closed.

(d) If A and B are two non-empty disjoint proper compact subsets of a Hausd space, show that there exists two disjoint open subsets G and H such t $A \subseteq G$ and $B \subseteq H$.