EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2009/2010

FIRST SEMESTER (June/July, 2011)

MT 101 - FOUNDATION OF MATHEMATICS

(REPEAT)

Answer all questions — Time : Three hours

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1. (a) Prove the following equivalences using the laws of algebra of propositions:

   i. $(p \wedge q) \vee \sim p \equiv \sim p \vee q$;

   ii. $(p \vee F) \wedge (q \vee \sim p) \wedge \sim (p \vee \sim q \vee T) \equiv F$;

   where F and T denote the statements which are always false and always true, respectively.

   (b) Using the valid argument forms, deduce the conclusion from the premises, giving reason for each step.

   $\sim p \vee q \rightarrow r$

   $s \vee \sim q$

   $\sim u$

   $p \rightarrow u$

   $\sim p \wedge r \rightarrow \sim s$

   $\therefore \quad \sim q$

2. (a) At least 70% of a class of students study Algebra, at least 75% students study calculus, at least 80% study Geometry and at least 85% study Trigonometry. What percentage (at least) must study all four subjects?

(b) For any set A, B and C, Prove that

i. $A \cup B = (A \Delta B) \cup (A \cap B)$;

ii. $A \cup B = (A \Delta B) \Delta (A \cap B)$;

iii. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$;

iv. $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

3. (a) Let $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x \neq 0,\ y \neq 0\}$ and define a relation R on S by $(x_1, y_1) R (x_2, y_2)$ iff $x_1 y_1 (x_2^2 - y_2^2) = x_2 y_2 (x_1^2 - y_1^2)$.

i. Show that R is an equivalence relation.

ii. If (a, b) is a fixed element of S, then show that $(x, y) R (a, b)$ iff $\frac{y}{x} = \frac{b}{a}$ or $\frac{y}{x} = \frac{-a}{b}$.

(b) Let ρ be a relation defined on $\mathbb{R}$ by $x \rho y$ iff $x^2 - y^2 = 2(2 - x)$.

i. Prove that ρ is an equivalence relation.

ii. Determine the $\rho-$ class of 1.

(c) Let R_1 and R_2 be equivalence relations on a set X.

i. Prove that $R_1 \cap R_2$ is an equivalence relation.

ii. Is $R_1 \cup R_2$ an equivalence relation? Justify your answer.

4. (a) Define each of the following terms:

i. injective mapping;

ii. surjective mapping;

iii. inverse mapping.

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x \mid x \mid$.

i. Show that f is a bijective function and determine f^{-1}.

ii. Is the mapping $g : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2 \mid x \mid$ a bijection? Justify your answer.

5. (a) Let $f : X \to Y$ be a mapping. Show that f is injective if and only if $f(A \cap B) = f(A) \cap f(B)$ for all subsets A, B of X.

(b) Show that every partially ordered set has at most one last element.

(c) Prove that the last element of every partially ordered set is a maximal element. Is the converse true? Justify your answer.

6. (a) State the *division algorithm.*

i. For any integer a, using division algorithm, prove that $3 \mid a(a+1)(a+2)$.

ii. For any odd integer a, show that $8 \mid (a^2 - 1)$.

(b) Using the Euclidean algorithm find the gcd(1819, 3587) and hence express the gcd(1819, 3587) as a linear combination of 1819 and 3587.

(c) A certain number of sixes and nines are added to give the sum of 126; if the number of sixes and nines are interchanged, the new sum is 114. How many of each were there originally?