EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
FIRST EXAMINATION IN SCIENCE -2009/2010
FIRST SEMESTER (June/July, 2011)
MT 151 - MATHEMATICA
(REPEAT)

Answer all Questions **Time: Two hours**

Attention: You are not allowed to access the facility of Mathematica **Help** option.

And you may use **Basic Input Palatte** for entering symbols **only**.

1. Use the Mathematica command(s) to solve the following problems.

   (a) Find the integer closest to $\sqrt{159}$.

   (b) Compute $1+\dfrac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{2}}}}$.

   (c) Compute a numerical approximation of $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{2}+\frac{1}{3}\right)\ldots\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{1}\right.$

   (d) Determine whether the expression $1+x\sin y+x^2\cos y+x^5e^y$ is a polynomial in x. Is it a polynomial in y?

   (e) Find the partial fraction expansion of $\dfrac{(x-1)^6}{(x^2+1)(x+1)^2(x-4)}$.

   (f) Plot the graph of $y=\sin x$ from 0 to π.

   (g) Sort the letters of the word "MISSISSIPPI" alphabetically.

   (h) Construct a 5×5 matrix having the first five primes as diagonal entries and zeros elsewhere.

   (i) Create a list contains all the subsets of $\{a,b,c,d,e\}$ which contain precisely three elements. How many are there?

(j) The 20^{th} prime is 71. Find all the numbers less than 71 which are not prime.

2. (a) The binomial coefficient $C(n,k) = \dfrac{n!}{k!(n-k)!}$ can be expressed as

$$\left(\frac{n}{k}\right)\left(\frac{n-1}{k-1}\right)\left(\frac{n-2}{k-2}\right)\cdots\left(\frac{n-k+1}{1}\right)$$

for more efficient computation. Use this representation to compute $C(10,4)$.

(b) The area enclosed by a triangle whose sides have length a, b and c is given by Heron's formula

$$K = \sqrt{s(s-a)(s-b)(s-c)},$$

where $s = \dfrac{a+b+c}{2}$. Express the area of a triangle as a function of a, b and c and compute the area of the triangle whose sides are 3, 4 and 5.

(c) If p dollars is invested for t years in a bank account paying an annual interest rate of r compounded n times per year, the amount of money after k periods is

$$p\left(1+\frac{r}{n}\right)^k$$

dollars. If 1000 dollars is invested in an account paying interest rate 6 compounded quarterly, make a table with suitable headings showing how much money is accumulated during a three year period.

(d) If x is an approximation to $\sqrt{a}$, then it can be shown that $\dfrac{1}{2}\left(x+\dfrac{a}{x}\right)$ is a better approximation. Use the command **NestList** to observe the first 10 approximation obtained in computing $\sqrt{3}$, starting with $x = 100$.

(e) The logistic equation for population growth given by $\dfrac{dp}{dt} = ap - bp^2$, was discovered in the mid-nineteenth century by the biologist Pierre Verhulst. The constant b is generally small in comparison to a so that for small population size p the quadratic term in p will be negligible and the population will grow approximately exponentially. For large p, however, the quadratic term serves to slow down the rate of growth of the population. Solve the logistic equation and sketch the solution for $a = 2$, $b = 0.005$, and an initial population $p_0 = 1$(thousand). Then determine the limiting value of the population as $t \to \infty$.