

Time: 01 hour.

Answer ALL Questions.

1. What do you understand by the terms macrostate, microstate and thermodynamic probability of a system. For a system obeying Maxwell Boltzman statistics, show that

$$a) E = \frac{Nk_B T^2}{Z} \frac{\partial Z}{\partial T}$$

$$b) S = Nk_B \ln Z + \frac{Nk_B T}{Z} \frac{\partial Z}{\partial T}$$

$$c) F = -Nk_B T \ln Z$$

Where the symbols have their usual meaning.



2. The thermodynamics probability of an assembly of Fermions can be written as

$$\Omega = \prod \frac{g_j!}{N_j!(g_j - N_j)!}$$

- a) If the system is in equilibrium, prove that

$$\sum_j \ln \left( \frac{N_j}{g_j - N_j} \right) dN_j = 0$$

- b) Also show that

$$\sum_j dN_j = 0 \text{ and}$$

$$\sum_j \varepsilon_j dN_j = 0$$

- c) Using the results in (a) and (b), obtain the Fermi-Dirac distribution function

- d) For a degenerate, spin  $\frac{1}{2}$ , non-interacting Fermi gas at zero temperature, show that the energy of a system of  $N$  such particles confined to a volume  $V$  can be written as

$$E = \frac{3N\varepsilon_F}{5}$$

Where the symbols have their usual meanings.