

EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE 2005/2006

FIRST SEMESTER (Aug./Sep.'2007)

ST 303 - REGRESSION ANALYSIS & QUALITY CONTROL
(Proper & Repeat)

Answer all questions

Time: Three hours

1. What is meant by "simple linear regression"? Distinguish between simple linear regression and multiple regression. State the method of least squares.
- (a) Estimate the simple linear regression parameters by the method of least squares.
- (b) Derive the maximum likelihood estimators of the above parameters.
2. Three water samples were taken at random at each of four depths in a river to determine whether the quality of dissolved oxygen varied from one depth to another. The data y_{ij} in the following table are dissolved oxygen for j^{th} sample ($j = 1, 2, 3$) at the i^{th} depth ($i = 1, 2, 3, 4$).

| Depth (x_i) | Dissolved Oxygen (y_{ij})' | \bar{y}_i |
|-----------------|--------------------------------|-------------|
| 1 | 4, 5, 6 | 5 |
| 2 | 6, 6, 6 | 6 |
| 3 | 7, 8, 9 | 8 |
| 4 | 8, 9, 10 | 9 |

With the usual notations,

$$S_{yy} = 36, \quad S_{xx} = 15, \quad S_{xy} = 21.$$

A simple linear regression model was proposed to predict dissolved oxygen.

$$y_{ij} = \alpha + \beta x_i + \epsilon_{ij}, \quad i = 1, 2, 3, 4, \quad j = 1, 2, 3, \quad \epsilon_{ij} \sim NID(0, \sigma^2)$$

- (a) Find the value of the least squares estimates for α and β . Give the fitted equation.
 - (b) Test the hypothesis $H_0 : \beta = 0$ Vs $H_1 : \beta \neq 0$ and give your conclusion (use $\alpha = 0.05$).
 - (c) Construct 95% confidence interval for β .
 - (d) Construct 95% confidence interval for the mean value of y at $x = 2$.
 - (e) Can a test for lack of fit be made here? Explain why you believe it can or cannot be made. If you believe that a test for lack of fit can be made, compute the statistics and state your conclusion.
3. (a) Give an example for a multiple linear regression with two independent variables.
- (b) For your example write down the model and the assumptions you make.
- (c) Consider the model:

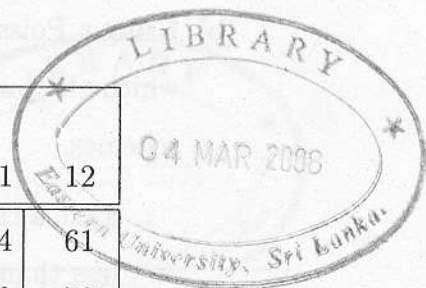
$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \epsilon.$$

It is given that there are 23 observations and $SST=39.2$, $SSR(X_1, X_2)=17.0$, $SSR(\text{on } X_1)=12.4$ and $SSR(\text{on } X_2)=5.2$.

- i. Construct the ANOVA table.
- ii. State the hypothesis that you will test using the ANOVA table and test the hypothesis at a significance level $\alpha = 0.05$.
- iii. Test whether it is necessary to include both X_1 and X_2 or one of them will do for prediction purposes.

4. Construct a control chart for \bar{X} and R for the following data on the basis of samples of fuses of 5, being taken every hour (each set of 5 has been arranged in ascending order of magnitude). Find the future control limits.

| Sample Number | | | | | | | | | | | |
|---------------|----|----|----|----|-----|-----|----|----|-----|-----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 42 | 42 | 19 | 36 | 42 | 51 | 60 | 18 | 15 | 69 | 64 | 61 |
| 65 | 45 | 24 | 54 | 51 | 74 | 60 | 20 | 30 | 109 | 90 | 78 |
| 75 | 68 | 80 | 69 | 57 | 75 | 72 | 27 | 39 | 113 | 93 | 94 |
| 78 | 72 | 81 | 77 | 59 | 78 | 95 | 42 | 62 | 118 | 109 | 109 |
| 87 | 90 | 81 | 84 | 78 | 132 | 138 | 60 | 84 | 153 | 112 | 136 |



5. The following table gives the number of errors observed at final inspection of a certain model of aeroplane. Prepare a C-Chart and comment on the picture. If the process does not seem to be in statistical control then revise the trial control limits.

| Aeroplane Number | Number of errors |
|------------------|------------------|
| 1 | 7 |
| 2 | 6 |
| 3 | 6 |
| 4 | 7 |
| 5 | 4 |
| 6 | 7 |
| 7 | 8 |
| 8 | 12 |
| 9 | 9 |
| 10 | 9 |
| 11 | 8 |
| 12 | 5 |
| 13 | 5 |

| Aeroplane Number | Number of errors |
|------------------|------------------|
| 14 | 9 |
| 15 | 8 |
| 16 | 17 |
| 17 | 6 |
| 18 | 4 |
| 19 | 13 |
| 20 | 7 |
| 21 | 8 |
| 22 | 17 |
| 23 | 6 |
| 24 | 6 |
| 25 | 10 |

6. (a) Give the basic concepts of Double Sampling Plan and state the advantages compared to Single Sampling Plan.
- (b) Using a Poisson approximation, find the probability of accepting a large batch which the proportion of defectives is $p = 0.01$ for each of the following sampling schemes.
- Take a random sample of size 100 and accept the batch if the sample contains less than 3 defectives, otherwise reject it.
 - Take a random sample of size 50. Accept the batch if it contains no defectives, otherwise reject the batch if it contains more than two defectives, otherwise take a second sample of size 100 and accept the batch only if the combined samples contain less than 4 defectives.
 - Determine the expected sample size for scheme (ii) and verify that it is less than the size of scheme (i).