EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE 2005/2006

FIRST SEMESTER(Aug./Sep.'2007)

ST 303 - REGRESSION ANALYSIS & QUALITY CONTROL

(Proper & Repeat)

Answer all questions

Time: Three hours

1. What is meant by "simple linear regression"? Distinguish between simple linear regression and multiple regression. State the method of least squares.

   (a) Estimate the simple linear regression parameters by the method of least squares.

   (b) Derive the maximum likelihood estimators of the above parameters.

2. Three water samples were taken at random at each of four depths in a river to determine whether the quality of dissolved oxygen varied from one depth to another. The data y_{ij} in the following table are dissolved oxygen for j^{th} sample $(j = 1, 2, 3)$ at the i^{th} depth $(i = 1, 2, 3, 4)$.

Depth (x_i)	Dissolved Oxygen (y_{ij})	$\overline{y}_i$
1	4, 5, 6	5
2	6, 6, 6	6
3	7, 8, 9	8
4	8, 9, 10	9

With the usual notations,

$$S_{yy} = 36, \qquad S_{xx} = 15, \qquad S_{xy} = 21.$$

A simple linear regression model was proposed to predict dissolved oxygen.

$$y_{ij} = \alpha + \beta x_i + \epsilon_{ij}, \qquad i = 1,\ 2,\ 3,\ 4, \qquad j = 1,\ 2,\ 3, \qquad \epsilon_{ij} \sim NID(0, \sigma^2$$

(a) Find the value of the least squares estimates for α and β. Give the fitted equ

(b) Test the hypothesis $H_0 : \beta = 0$ Vs $H_1 : \beta \neq 0$ and give your conclusion (use $\alpha = 0.05$).

(c) Construct 95% confidence interval for β.

(d) Construct 95% confidence interval for the mean value of y at $x = 2$.

(e) Can a test for lack of fit be made here? Explain why you believe it can or ca be made. If you believe that a test for lack of fit can be made, compute the statistics and state your conclusion.

3. (a) Give an example for a multiple linear regression with two independent varia

(b) For your example write down the model and the assumptions you make.

(c) Consider the model:

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \epsilon.$$

It is given that there are 23 observations and SST=39.2, SSR(X_1, X_2)=17.6 SSR(on X_1)=12.4 and SSR(on X_2)=5.2.

i. Construct the ANOVA table.

ii. State the hypothesis that you will test using the ANOVA table and test hypothesis at a significance level $\alpha = 0.05$.

iii. Test whether it is necessary to include both X_1 and X_2 or one of them w do for prediction purposes.

4. Construct a control chart for $\overline{X}$ and R for the following data on the basis of samples of fuses of 5, being taken every hour (each set of 5 has been arranged in ascending order of magnitude). Find the future control limits.

Sample Number											
1	2	3	4	5	6	7	8	9	10	11	12
42	42	19	36	42	51	60	18	15	69	64	61
65	45	24	54	51	74	60	20	30	109	90	78
75	68	80	69	57	75	72	27	39	113	93	94
78	72	81	77	59	78	95	42	62	118	109	109
87	90	81	84	78	132	138	60	84	153	112	136

5. The following table gives the number of errors observed at final inspection of a certain model of aeroplane. Prepare a C-Chart and comment on the picture. If the process does not seem to be in statistical control then revise the trial control limits.

Aeroplane Number	Number of errors
1	7
2	6
3	6
4	7
5	4
6	7
7	8
8	12
9	9
10	9
11	8
12	5
13	5

Aeroplane Number	Number of errors
14	9
15	8
16	17
17	6
18	4
19	13
20	7
21	8
22	17
23	6
24	6
25	10

6. (a) Give the basic concepts of Double Sampling Plan and state the advantages compared to Single Sampling Plan.

(b) Using a Poisson approximation, find the probability of accepting a large batc which the proportion of defectives is $p = 0.01$ for each of the following samp schemes.

i. Take a random sample of size 100 and accept the batch if the sample cont less than 3 defectives, otherwise reject it.

ii. Take a random sample of size 50. Accept tha batch if it contains no defect reject the batch if it contains more than two defectivs, otherwise take a se sample of size 100 and accept the batch only if the combined samples co less than 4 defectives.

iii. Determine the expected sample size for scheme (ii) and verify that it is than the size of scheme (i).