EASTERN UNIVERSITY, SRI LANKA
THIRD EXAMINATION IN SCIENCE - 2004/2005
SECOND SEMESTER(Dec.,2008/Jan.,2009)
MT 307 - CLASSICAL MECHANICS III
(SPECIAL REPEAT)

Answer all Questions Time: Three hours

Q1. (a) With the usual notation, for a rotating system of axis, prove that

$$\frac{d^2\underline{r}}{dt^2} = \frac{\partial^2\underline{r}}{\partial t^2} + \frac{\partial \underline{w}}{\partial t} \wedge \underline{r} + 2\underline{w} \wedge \frac{\partial \underline{r}}{\partial t} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r}).$$

(b) If a particle of mass m is projected vertically upward with velocity v and latitude λ, show that after time T it will strike the earth at a distance

$$\frac{4wv^3}{3g^2} \cos\lambda,$$

where ω is the angular velocity of the earth and g is the gravitational acceleration, along westward direction from its starting point.

Q2. (a) State the linear momentum principle.

With the usual notation, show that

$$\sum_{i=1}^{n} (\underline{r}_i - \underline{r}_A) \wedge \underline{F}_i = (\underline{r}_G - \underline{r}_A) \wedge M\underline{f}_G + \frac{d\underline{H}_G}{dt}.$$

(b) A solid of mass M is in the form of a tetrahedron $OXYZ$, the edges OX, OY and OZ of which are mutually perpendicular, rests with XOY on a fixed smooth horizontal plane and YOZ against a smooth vertical wall. The normal to the rough face XYZ is in the direction of a unit vector $\underline{n}$. A heavy unifo sphere of mass m and center C rolls down the face causing the tetrahedron acquire a velocity $-V\underline{j}$, where j is the unit vector along OY. If $\overrightarrow{OC} = \underline{r}$, pro that

$$(\dot{M} + m)V - m\dot{\underline{r}}.\underline{j} = l,$$

where l is a constant, and

$$\frac{7}{5}\ddot{\underline{r}} = \underline{f} - \underline{n}(\underline{n}.\underline{f}),$$

where $\underline{f} = \underline{g} + \dot{V}\underline{j}$ and g is the gravitational acceleration.

Q3. (a) Derive Euler's equations of motion of a rigid body with one point fixed.

(b) A solid consist of two uniform right circular cones which are rigidly joined the vertex O such that their axis in the same straight line with the vert angle $\frac{\pi}{2}$. The height of each cone is b. If O is fixed and the solid is set rotate about a common generator of the cone with angular velocity ω, under forces except gravity and reaction at O, show that the solid will rotate abo the same generator after a time $\frac{10\sqrt{2}\,\pi}{3\omega}$.

Q4. (a) Write down the equation of D'Alembert's principle and virtual work. Hen obtain Lagrange's equation for a Holonomic system.

(b) Find differential equations of motion for a spherical pendulum of length l.

Q5. (a) A uniform rod AB of length $2l$ and mass m has a particle of mass M attache to the end B. The system is at rest on a smooth horizontal table. An impul I is applied to A in a direction perpendicular to AB in the plane of the tabl Find the initial velocities of A and B and prove that the resulting kinetic ener is

$$\frac{2I^2(m + 3M)}{m(m + 4M)}.$$

(b) If f and g of dynamical variables $\vec{p}, \vec{q}$ and time t are constant functions, pro that it's Poisson bracket is also constant of the motion.

Q6. Consider a system consists of two identical simple pendula each of mass m, length and coupled by a massless spring of force constant k. They move in a vertical plan and the two pendula are identical in an equilibrium position. If a small horizonta oscillation about the potion of equilibrium is concerned, then

(a) find Lagrangian function and

(b) show that the horizontal displacements of the pendula are given by

$$\alpha e^{it\omega_o} + \beta e^{it\sqrt{\omega_0^2+2\omega_s^2}} \quad \text{and} \quad \alpha e^{it\omega_o} - \beta e^{it\sqrt{\omega_0^2+2\omega_s^2}},$$

where

$$\omega_0 = \sqrt{\frac{g}{l}} \quad \text{and} \quad \omega_s = \sqrt{\frac{k}{m}}.$$