

## EASTERN UNIVERSITY, SRI LANKA DEPARTMENT OF MATHEMATICS SECOND EXAMINATION IN SCIENCE - 2009/2010

FIRST SEMESTER (June/July, 2011)

## MT207 - NUMERICAL ANALYSIS

· (Proper & Repeat)

Answer all questions

Time: Two hours

LIB

- 1. (a) Define the following terms:
  - i. absolute error;
  - ii. relative error of a numerical value.
  - (b) Evaluate  $f(x) = x^3 6x^2 + 3x 0.149$  at x = 4.71 using three digit arithmetic with chopping. Compute the absolute error and relative error.
  - (c) Repeat the calculation in part (b), using the nesting form of f(x) that was found in part (b). Calculate the relative error and compare with that found in part (b).
  - (d) Describe what is meant by truncation error by reference to approximating  $\sin x$  by x.

2. (a) Let x = g(x) is the rearrangement of the equation f(x) = 0 and define the iteration,

$$x_{n+1} = g(x_n);$$
  $n = 0, 1, 2...$  (1)

with the initial value  $x_0$ . If g'(x) exists, is continuous, and  $|g'(x)| \leq K < 1$  for all x, then show that the sequence  $(x_n)$  generated by the iteration (1) converges to the unique root  $\alpha$  of the equation f(x) = 0.

Show that the iteration,  $x_{i+1} = \frac{2x_i - 3}{2 - x_i}$ , have fixed points at  $x = \pm \sqrt{3}$ .

Hence investigate the convergence of the method.

(b) Obtain the Newton Raphson algorithm to compute the roots of the equations f(x) = 0 in an interval [a, b].

Sketch the cubic polynomial  $f(x) = 4x^3 - 10x^2 + 2x + 5$  to get a rough estimate of its roots. Use the Newton Raphson method to approximate each root to four decimal places.

3. (a) Construct a forward difference table for the following data.

x.	1.0	1.5	2.0	2.5
f(x)	0.8988	0.9613	0.9945	0.9976

With  $x_0 = 0.1$ , estimate the approximation for the first derivative of f(x) at x = 1.5 using the Newton's forward formula.

(b) Obtain the composite Trapezoidal rule to estimate  $\int_a^b f(x)dx$  and derive a formula for error.

Let

$$I = \int_0^1 e^{-x^2} dx.$$

Estimate I using the composite Trapezoidal rule with 10 sub-intervals. Find an error bound in the elimination.

4. (a) Solve the following system of linear equations using the Gaussian Elimination with two digit rounding arithmetic and partial pivoting:

$$2x_1 + 4x_2 - x_3 = -5,$$

$$x_1 + x_2 - 3x_3 = -9,$$

$$4x_1 + x_2 + 2x_3 = 9.$$

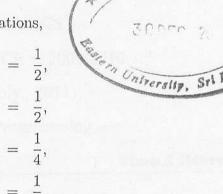
(b) Find the solution of the following system of equations,

$$x_1 - \frac{1}{4}x_2 - \frac{1}{4}x_3 = \frac{1}{2},$$

$$-\frac{1}{4}x_1 + x_2 - \frac{1}{4}x_4 = \frac{1}{2},$$

$$-\frac{1}{4}x_1 + x_3 - \frac{1}{4}x_4 = \frac{1}{4},$$

$$-\frac{1}{4}x_2 - \frac{1}{4}x_3 + x_4 = \frac{1}{4},$$



using the Gauss-Seidel method and perform the first three iterations.