EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2007/2008

SECOND SEMESTER(December/January, 2008/2009)

ST 104 - DISTRIBUTION THEORY

(SPECIAL REPEAT)

Answer all Questions

Time: Three hours

Q1. (a) If U has a χ^2 distribution with n degrees of freedom,

$$\theta = \begin{cases} \dfrac{e^{-u/2}\ u^{(\frac{n}{2}-1)}}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})} & u \neq 0; \\ 0, & \text{otherwise.} \end{cases}$$

Find $E(U)$ and $V(U)$.

(b) Let $Y_1, Y_2, Y_3, ..., Y_n$ be random sample from a normal distribution with mean μ and variance σ^2. Find the $E(S^2)$ and $V(S^2)$, where

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}[Y_i - \overline{Y}]^2 \text{ and } \overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

Q2. Let $X \sim U[0,1]$ and $Y \setminus (X = x) \sim B(n, x)$ i.e.,

$$P(Y = y \setminus X = x) = \binom{n}{y} x^y (1-x)^{n-y},\ y = 0, 1, 2, \ldots, n.$$

Find the distribution of Y. Also find $E(Y)$ and $V(Y)$.

Q3. (a) A particular fast-food outlet is interested in the joint behavior of the random variables Y_1, defined as the total time between a customer's arrival at the store and leaving the service window, and Y_2, the time that a customer waits in line before reaching the service window. Because Y_1 contains the time a customer waits in line, we must have $Y_1 \geq Y_2$. The relative frequency distribution of observed values of Y_1 and Y_2 can be modeled by the probability density function

$$f(y_1, y_2) = \begin{cases} e^{-y_1} \;; & 0 \leq y_2 \leq y_1 < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

Another random variable of interest is $U = Y_1 - Y_2$, the time spent at the service window.

(i) Find the probability density function for U.

(ii) Find $E(U)$ and $V(U)$.

(b) Let $Y_1, Y_2, ..., Y_n$ be independent uniformly distributed random variables on the interval $[0,\ \theta]$.

(i) Find the probability distribution function of $Y_{(n)} = max(Y_1, Y_2, ..., Y_n)$.

(ii) Find the density function of Y_n.

(iii) Suppose that the number of minutes that you need to wait for a bus is uniformly distributed on the interval [0,15]. If you take the bus five times what is the probability that your longest wait is less than 10 minutes?

Q4. (a) The joint distribution for the length of life of two different types of components operating in a system was given in

$$f(y_1, y_2) = \begin{cases} \frac{1}{8} y_1 e^{\frac{-(y_1+y_2)}{2}} \;; & y_1 > 0,\ y_2 > 0, \\ 0 & \text{otherwise.} \end{cases}$$

The relative efficiency of the two types of components is measured by $U = Y_2/Y_1$. Find the probability density function for U.

(b) Two efficiency experts take independent measurements Y_1 and Y_2 on the length of time it takes workers to complete a certain task. Each measurement is assumed to have the density function given by

$$f(y) = \begin{cases} \frac{1}{8} y_1 e^{\frac{-y}{2}} \;; & y > 0,, \\ 0 & \text{otherwise.} \end{cases}$$

By using moment generating functions, find the density function for the average $U = (1/2)(Y_1 + Y_2)$

Q5. Let Y be a random variable with density function given by

$$f(y) = \begin{cases} \frac{3}{2}y^2, & -1 \leq y \leq 1; \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the density function of $U_1 = 3Y$.

(b) Find the density function of $U_2 = 3 - Y$.

(c) Find the density function of $U_3 = Y^2$.

(d) Find $V(U_1)$, $V(U_2)$ and $V(U_3)$

Q6. (a) Let Y_1 and Y_2 be independent and uniformly distributed over the interval (0,1). Find the probability density function of the following:

(i) $U_1 = \min(Y_1, Y_2)$

(ii) $U_1 = \max(Y_1, Y_2)$

(b) Candidate A believes that he can win a city election if he can poll at least 55% of the votes in Precinct I. He also believes that about 50% of the city's voters favor him. If n=100 voters show up to vote at Precinct I, what is the probability that candidate A receives at least 55% of the votes?