30 DEC 2011
Eastern University, Sri Lanka

EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
THIRD EXAMINATION IN SCIENCE 2009/2010
FIRST SEMESTER (June/July, 2011)
MT 304 - GENERAL TOPOLOGY

Answer all questions Time : Two hours

1. Define the following terms:

 - Topology on a set;
 - Subspace of a topology.

 (a) Let X be a non-empty set and τ be the collection of subsets of X consisting the empty set Φ and all subsets of X whose complements are finite. Is (X, τ) a topological space? Justify your answer.

 (b) Let (Y, τ_Y) be a subspace of a topological space (X, τ). Prove that $A \subseteq Y$ is closed in (Y, τ_Y) if and only if, $A = F \cap Y$ for some closed subset F of X in (X, τ).

 (c) Let $\mathbb{B}$ be a base for a topology τ on X and $S \subseteq X$. Prove that the collection $\mathbb{B}_S = \{U \cap S \mid U \in \mathbb{B}\}$ is a base for the subspace topology τ_S on S of (X, τ).

2. Define the *closure* of a subset A in a topological space (X, τ)

(a) Let A be a non-empty subset of a topological space (X, τ). Prove that the closure of A is the smallest closed set containing the set A.

(b) Let A be a subset of a topological space (X, τ),where X is an infinite set and $\tau = \{A \subseteq X \mid A = \phi \text{ or } A^C \text{ is finite}\}$. Prove that the set of all limit points of A is closed.

(c) Let f be a continuous function from a topological space (X, τ) into (Y, τ'). Prove that for every subset A of X, $f(\overline{A}) \subseteq \overline{f(A)}$, where $\overline{A}$ is the closure of A.

3. (a) If (X, τ) is a compact space and A is a closed subset of X, then prove that A is compact.

(b) Prove that the continuous image of a compact set in a topological space is compact.

(c) Prove that every compact subset of a Hausdorff topological space is closed.

(d) If f is a bijective, continuous function from a topological space (X, τ) to a topological space (Y, τ') and Y is a Hausdorff space, then prove that f is a homeomorphism.

4. (a) Prove that a topological space (X, τ) is a Frechet space if and only if, every singleton subset of X is closed.

(b) Prove that a topological space (X, τ) is disconnected if and only if, there exists a non-empty proper subset of X, which is both open and closed.

(c) Let (X, τ) and (Y, τ') be two topological spaces and $f : X \to Y$ be a continuous function. If A is a connected subset of X, then prove that the image of A is connected.