EASTERN UNIVERSITY, SRILANKA
DEPARTMENT OF MATHEMATICS

THIRD EXAMINATION IN SCIENCE - 2009/2010
FIRST SEMESTER (June /July, 2011)
MT 306 – PROBABILITY THEORY

Answer all questions **Time: 2 Hours**

(01) Let X have an exponential distribution with parameter λ, so that its probability density function (pdf) is

$$f(x)=\lambda e^{-\lambda x}, \ 0 \leq x$$

(i) Show that the moment generating function(mgf) $M_X(t)$ of X is

$$M_X(t)=\frac{\lambda}{\lambda - t}, \quad t < \lambda.$$

(ii) Use $M_X(t)$ to find the mean and variance of X.

(iii)Let $Y = 3X + 1$: Find the mgf of Y. State, with reasons, whether or not Y has an exponential distribution.

(02) (a) The weight of a certain brand of chocolate bars are assumed to be normally distributed with $\mu=50\,g$ and standard deviation $\sigma=1g$. A random sample of 7 bars is taken. Find the probability that weight of a bar lies between 49g and 52g. Further, chocolate bars having less than 40g are not assumed to be in the standard quality in weight. Find the probability a chocolate bar has the standard quality in weight.

(b) Seats of a boat service, provide by a certain person should be booked early. In a boat, the maximum number of passengers can be carried is 10. To cover the expenses of one ride, at least 3 passengers should attend for the boat ride. Probability that a person who has booked a seat, will attend is 0.7. Find the probability that this service provider gets a loss from a certain boat ride.

(03) Let $f_{XY}(x,y)$ be the 2-dimentional density and it is given by

$$f_{XY}(x,y)=\begin{cases} c\,e^{-\lambda x} & ; 0<y<x \\ 0 & ; \text{else} \end{cases}$$

(a) Find the constant c.

(b) Find the marginal densities $f_X(x)$ and $f_Y(y)$.

(c) Use $f_X(x)$ to find E(X) and V(X).

(d) Find the conditional distribution of X given Y.

(e) Find the probability Pr($X > 2 / Y=1$).

(04) (a) It is assumed that number of accident occur in a certain city, has a Poisson distribution with parameter λ.

(i) Use the method of moment and maximum likelihood separately to find an estimator for parameter λ.

(ii) Are they unbiased estimators for λ? Justify your answers. If not, find their biases.

(b) Assume $X_1, X_2, X_3, \ldots, X_n$ be a random sample obtained from a normal distribution having mean μ and known variance σ^2. Derive a (1-α)100% confidence interval for this population mean μ. Use the following sample to find 95% confidence limits of μ. (Sample: 10, 15, 12, 16, 14, 15, 20, 25, 11, 12)

(Assume $\sigma^2 = 25$ and $Z_{0.05} = 1.64$, $Z_{0.025} = 1.96$, $Z_{0.95} = -1.65$)

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