EASTERN UNIVERSITY, SRI LANKA
SECOND YEAR/SECOND SEMESTER EXAMINATION IN SCIENCE (2002/03 & 2002/03(A))

(April./May.'2004)

MT 218 - FIELD THEORY

(REPEAT)

Answer all questions **Time:Two hours**

1. State and prove Gauss's theorem.

   (a) Assuming that the total charge Q of an atomic nucleus is uniformly distributed within a sphere of radius $'a'$, show that the potential at a distance r from the center $(r \leq a)$ is ;

   $$V = \left\{3 - \left(\frac{r}{a}\right)^2\right\} \frac{Q}{8\pi\epsilon_0 a}$$

   (b) A charge q is uniformly distributed on a circle with equations $x^2 + z^2 = a^2,\ y = 0$. Show that the potential at a point $P(0, y, 0)$ is $\dfrac{q}{4\pi\epsilon_0\sqrt{a^2+y^2}}$.

   Prove also that the electric field at P is $\dfrac{qy\mathbf{j}}{4\pi\epsilon_0\sqrt{\{a^2+y^2\}^3}}$ where $\underline{j}$ is the unite vector along the Y axis.

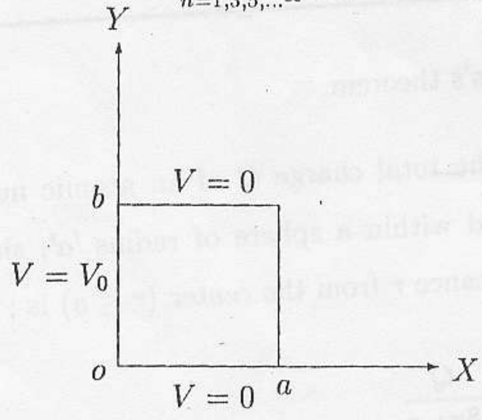
2. Show by using separation of variable or otherwise, that the solution of the equation $\nabla^2 V = 0$, where V is the potential function in two dimensional rectangular co-ordinates is given by;

$$V = (A_1 \sinh \alpha x + A_2 \cosh \alpha x)\,(B_1 \sin \alpha y + B_2 \cos \alpha y)$$

where $A_1, A_2, B1, B_2$ and α are arbitrary constants.

Prove that the potential distribution inside the rectangular region shown in figure,for the boundary conditions noted is given by;

$$V(x,y) = \sum_{n=1,3,5,\ldots\infty} \frac{4V_0}{n\pi} \sin \frac{n\pi y}{b} \frac{\sinh \frac{n\pi}{b}(a-x)}{\sinh\,(\frac{n\pi a}{b})}$$



$c-i$	$V = 0$	at	$y = 0, y = b$	$0 \leq x \leq a$
$c-ii$	$V = 0$	at	$x = a$	$0 \leq y \leq b$
$c-iii$	$V = V_0$	at	$x = 0$	$0 \leq y \leq b$

$$\left(Hint: \int_0^b \sin \frac{n\pi y}{b} \sin \frac{m\pi y}{b} dy = \begin{cases} 0 & \text{if } n \neq m \\ \frac{b}{2} & \text{if } n = m, \; n \neq 0 \end{cases} \right)$$

3. (a) Define the term "electric dipole".

Prove that the electric potential V at a point P at a distance r form the dipole of moment $\underline{P}$ is given by

$$V = -\frac{1}{4\pi\varepsilon_0}\left\{\underline{P}\cdot grad\left(\frac{1}{r}\right)\right\}.$$

Hence prove that the force on a dipole in an electric field E is given by,

$$\underline{F} = (\underline{P}\cdot\nabla)\underline{E}$$

(b) What is dielectric polarization ?

Show, with the usual notation that the potential due to a finite volume of dielectric is given by

$$V = \frac{1}{4\pi\epsilon_0}\int_s \frac{\underline{P}.d\underline{s}}{r} + \frac{1}{4\pi\epsilon_0}\int_\tau \frac{-\text{div}\underline{P}}{r}d\tau$$

Interpret this result.

4. (a) Define the magnetic flux density $\underline{B}$ and show that $div\underline{B} = 0$ in space.

By assuming the Ampere's law in integral form deduce the equation $curl\ \underline{B} = \mu_0\underline{j}$ where $\underline{j}$ is the current density.

(b) Define the magnetic field strength $\underline{H}$ in a magnetizable media and show that $curl\underline{H} = \underline{j}$.

If no currents are present and the magnetization is linearly proportional to $\underline{H}$, show that there exists a function ϕ such that $\nabla^2\phi = 0$.

(c) Prove that the magnetic field at a distance d from an infinitely long straight wire which carries current I, is given by $\frac{\mu_0}{2\pi}\left(\frac{I}{d}\right)$ Where μ_0 is the magnetic permeability of vacuum.