EASTERN UNIVERSITY, SRI LANKA

SPECIAL DEGREE EXAMINATION IN SCIENCE – 2009/10

FIRST SEMESTER

(May 2010)

PH 402 ADVANCED QUANTUM MECHANICS

Time: 03 Hours.

Answer ALL Questions

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1. A particle is described at time $t = 0$ by the wave packet

$$\psi(x) = \int_{-\infty}^{+\infty} A(k)\exp(ikx)\,dk$$

With, $A(k) = \begin{cases} A_0/k_0 & \text{for } 0 \le k \le k_0 \\ 0 & \text{for other } k \text{ values} \end{cases}$

(a) Show that the wave packet satisfies:

$$|\psi(x)|^2 = |A_0|^2 \frac{\sin^2\left(\frac{k_0 x}{2}\right)}{\left(\frac{k_0 x}{2}\right)^2}$$

(b) The uncertainty Δx in the position of the particle at time $t = 0$ is defined by the smallest positive value of x for which the function $|\psi(x)|^2$ obtained in (a) is zero. Show that this is given by

$$\Delta x = \frac{2\pi}{k_0}$$

(c) The uncertainty in the momentum Δp_x of the particle at tim
$t = 0$ is given by

$$\Delta p_x = hk_0$$

Show that this uncertainty in momentum, together with th
uncertainty in position obtained in (b), satisfy the Heisenber
uncertainty principle.

Here the symbols have their usual meaning.

2. The one dimensional time-independent Schrödinger equation fo
a particle of mass m moving in a potential $V(x)$ is given by

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right)u(x)=Eu(x),$$

where E is the total energy. A potential barrier is defined by

Region 1	$x<0$ and $V(x)=0$
Region 2	$0\le x\le a$ and $V(x)=V_0$
Region 1	$x>a$ and $V(x)=0$

where $V_0 > 0$ and $a > 0$.

(i) Consider the case where particles are incident on the barrier from region 1 with total energy $E > V_0$. Demonstrate that the Schrödinger equation has the following solutions in three regions,

$$u_1(x)=e^{ikx}+Ae^{-ikx}$$
$$u_2(x)=Be^{iqx}+Ce^{-iqx}$$
$$u_3(x)=De^{ikx}$$

where $k^2=\frac{2mE}{\hbar^2}$, $q^2=\frac{2m(E-V_0)}{\hbar^2}$ and A, B, C and D are constants.

3.

(ii) What is the significance of the two terms in the solution in Region 1 and why there is no term e^{-ikx} for $x > a$?

(iii) State the continuity conditions that must be satisfied by the wave function at $x = 0$ and $x = a$.

(iv) In general, some particles are reflected by the barrier, but for particular values of the energy $E > V_0$, all particles are transmitted and none are reflected, so that $A = 0$. By using the continuity conditions at $x = 0$, show that when total transmission occurs,

$$B = \frac{(q+k)}{2q} \text{ and } C = \frac{(q-k)}{2q}.$$

(v) Use the continuity conditions at $x = a$ to show that total transmission occurs for energies for which $qa = n\pi$, where $n = 1, 2, 3, \ldots\ldots$

3. (a) For any pair of allowed wave functions of the quantum system under discussion, if we define the Hermitian conjugate $\hat{F}^{+}$ of an operator $\hat{F}$ as

$$\left(\psi_1, \hat{F}^{+}\psi_2\right) = \left(\psi_2, \hat{F}\psi_1\right)^{*}$$

Show that

(i) If $\hat{F}$ is a Hermitian, then $\hat{F}^{+} = \hat{F}$

(ii) $\left(\hat{F} + i\hat{G}\right)^{+} = \hat{F}^{+} - i\hat{G}^{+}$

(iii) $\left(\hat{F}\hat{G}\right)^{+} = \hat{G}^{+}\hat{F}^{+}$

(iv) If $\phi = \hat{F}\psi$, then $(\phi, \psi) = \left(\psi, \hat{F}^{+}\psi\right)$

(b) If $\hat{R}_+$ and $\hat{R}_-$ are the ladder operators introduced in the soluti of the one dimensional simple harmonic oscillator proble show that

$$\frac{1}{2m}\hat{P}^2 = \frac{1}{4}\left(\hat{R}_+^2 + \hat{R}_-^2\right) + \frac{1}{2}\hat{H}$$

here the symbols have their usual meaning.

Hence show that the expectation value of the kinetic energy the oscillator when it is in a state of definite energy is one-ha of the total energy.

4. The radial part of a schrodinger equation of the hydrogen ato can be written as

$$\frac{d^2R(r)}{dr^2} + \frac{2}{r}\frac{dR(r)}{dr} + \frac{2m}{\hbar^2}\left(E + \frac{e^2}{4\pi\varepsilon_0 r}\right) - \frac{l(l+1)}{r^2}R(r) = 0$$

here the symbols have their usual meanings

(i) Find the ground state eigen function of the form $R(r) = Ae^{-}$ and the energy of the electron in this state, where A is constant and $a = \frac{4\pi\varepsilon_0\hbar^2}{me^2}$.

(ii) At the ground state show that the probability of finding th electron is maximum for $r = a$.

(iii) Find the expectation value of r.

(iv) Show that the expectation value of the potential energy o the electron is twice the energy at the ground state.

You may assume that

$$\int_0^{+\infty} r^n e^{-\frac{r}{a}} dr = n!\,a^{n+1}$$

6. (a) Explain briefly the meaning of perturbation. Find the fi order time independent perturbation correction to the ener and the wave function for non-degenerate levels.

(b) A particle of mass m bound by the harmonic oscillat potential $V(x)=\frac{1}{2}m\omega^2x^2$, is in its ground state. A we perturbation now adds to the potential an amount λ

Calculate the new ground state energy to first order λ.

Assume that the ground state wave function for the harmon oscillator is given by $\psi_0 = \left(\frac{m\,\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\left(\frac{m\,\omega\,x^2}{2\hbar}\right)}$.