EASTERN UNIVERSITY, SRI LANKA

SPECIAL DEGREE EXAMINATION IN SCIENCE – 2009/10

FIRST SEMESTER

(May 2010)

PH 403 CLASSICAL MECHANICS

me: 01 Hour.

nswer ALL Questions

A particle of mass m with energy E and angular momentum l, moves in a plane under the action of central field with potential $V = kr^{1/2}$ where k is a constant ($k>0$).

(a) Show that the angular momentum is a constant.

(b) Show that the motion of the particle is equivalent to that of the motion of a particle in one dimension in an effective potential given by:

$$V_{eff} = \frac{l^2}{2mr^2} + kr^{1/2}$$

(c) Obtain expressions for the energy and angular momentum of the particle if it moves in a circular orbit of radius a.

(d) If a small displacement is given to the particle radially while the particle moving in the circular orbit. Show that the particle executes simple harmonic oscillation about the circular orbit with angular frequency:

$$\sqrt{\frac{5k}{4ma^{3/2}}}.$$

(e) A planet of mass m is orbiting the Sun of mass M in the gravitationa of gravitational constant G. At perihelion, the magnitude L of ar momentum is $mv(1-\varepsilon)a$ and at aphelion it is $mv'(1+\varepsilon)a$. At perih the energy E is $\frac{mv^2}{2} - \frac{GMm}{a(1-\varepsilon)}$ and at aphelion it is $\frac{mv^2}{2} - \frac{Gl}{a(1}$ Where v and v' are the velocities of the planet at perihelior aphelion of the elliptical orbit of the planet respectively. The paran a and ε are the radius of the semi major axis and the eccentricity elliptical orbit of the planet respectively.

(i) Using conservation of angular momentum deduce $v = \frac{L}{m(1-\varepsilon)a}$ and $v' = \frac{L}{m(1+\varepsilon)a}$.

(ii) Using conservation of energy, deduce $L^2 = GMm^2a(1-\varepsilon^2)$

(iii) Using the formula for L^2, deduce that $E = -\frac{GMm}{2a}$

(a)

(b)

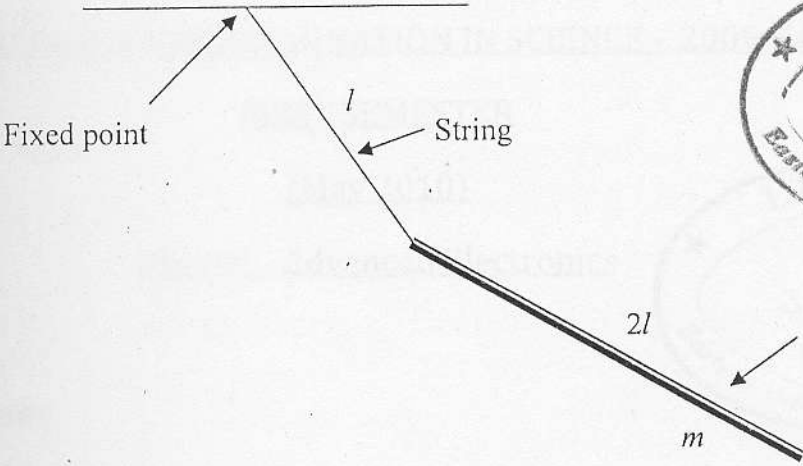
(c)

(d) Explain what is meant by normal coordinates and normal mode vibrations of a system.

(e) One end of an inextensible string of length l is attached to one end uniform rod of length $2l$ and mass m and the other end of the strin attached to a fixed point. The system hangs vertically in the equilibr position. A small displacement of the rod is made from the equilibr position so that the string remains taught during the displacement. configuration of the system may be specified by the angle θ and ϕ mad the string and rod respectively with the downward vertical.

(f)

(g)



(a) Write down the kinetic and potential energies of the system and show that the Lagrange's function of the motion is given by:

$$L = \frac{1}{2}ml^2\dot{\theta}^2 + \frac{2}{3}ml^2\dot{\phi}^2 + ml^2\dot{\theta}\dot{\phi}\cos(\theta - \phi) + mg(l\cos\theta + l\cos\phi).$$

(b) Assuming the oscillations are small, deduce the Lagrangian function for small oscillations about the equilibrium position.

(c) Derive an expression for the Hamiltonian function from the Hamiltonian function of the system for small oscillations.

(d) Obtain the Hamilton's equations for the system. Hence deduce the equations of motion of the system.

(e) Show that the normal frequencies of vibrations of the system are $\sqrt{\frac{g}{l}\left(\frac{6}{7+\sqrt{37}}\right)}$ and $\sqrt{\frac{g}{l}\left(\frac{6}{7-\sqrt{37}}\right)}$.

(f) Write down the general solution for the equations of motion.

(g) Find the normal coordinates of the system.