EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
SPECIAL DEGREE EXAMINATION IN COMPUTER SCIENCE (2008/2009)
(PART I)
CS 405 - THEORY OF COMPUTATION

Answer all questions **Time allowed: 3 hours**

1) Consider the three languages i, ii, iii below, over the alphabets $\{a,b\}$.

(a) Decide where each fit into in Fig. 1, and justify your decision.

(b) For each of the languages i, ii, & iii,

- If you decide that a language L is regular, give either a regular expression or a regular grammar for it, **and** design a finite automaton to accept it.
- If a language is context-free but not regular, show that it is not regular, **and** give either a context-free grammar or pushdown automaton for it.
- If a language is recursive but not context-free, show that it is not context-free, and give a Turing machine that accepts the language and halts on every input.

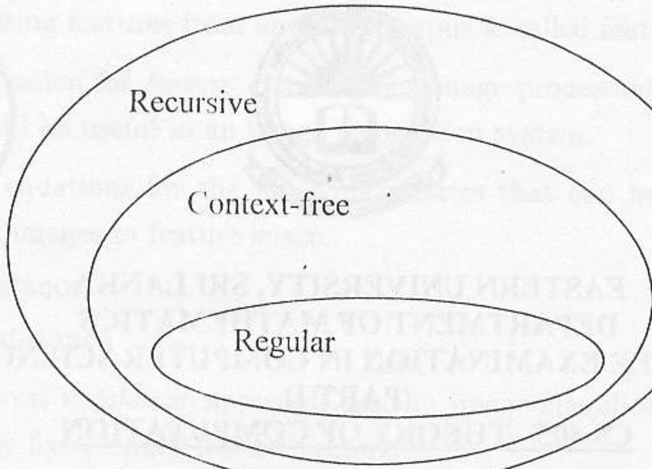

Fig.1

i. $\{a^n b^n a^n : n \geq 0\}$

ii. $\{a^n b^m a^k : n, m, k \geq 0\}$

iii. $\{a^n b^n a^m : n, m \geq 0\}$.

2)

(a) Show by induction that if a set X has n elements then the power set $P(X)$ of X has 2^n elements.

(b) Give a recursive definition of the language **Equal** consisting of all strings over $\{a, b\}$ having an equal number of a's and b's. Use concatenation as operator.

(c) Prove that $P(N)$, the set of all subsets of N, the set of all natural numbers, is uncountable.

(d) Prove that 3 is a factor of $n^3 - n + 3$ for all $n \geq 0$.

3)

(a) Define a Deterministic Finite Automation (DFA) that recognizes the set of all strings over $\{0, 1\}$ that start with 0 and has odd length, or start with 1 and has even length.

(b) Define a Deterministic Finite Automation (DFA) that recognizes the following language over $\{0, 1\}$:

$L = \{w : \text{w has '1' on every odd position and } w \neq \lambda\}$

(c) What is the language recognized by the following DFA in Fig. 2?

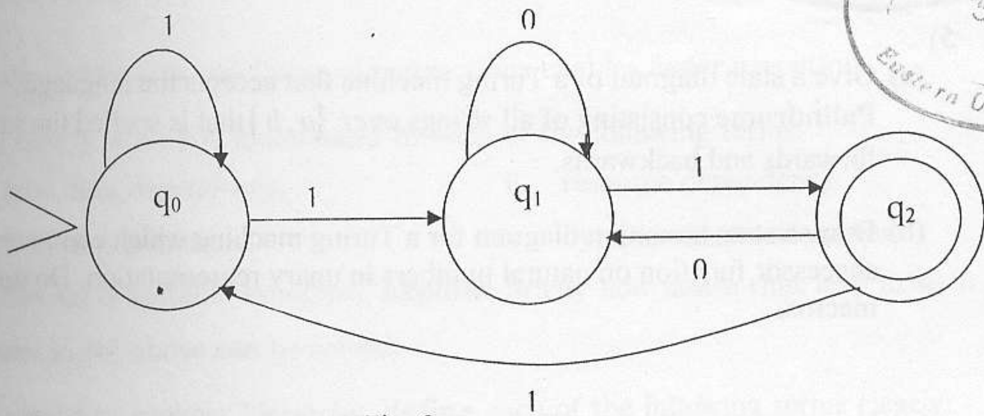

Fig. 2

(d) Let L be the set of all non-null strings over the alphabets $\sum = \{1,3\}$ whose character wise sum up to a multiple of 5. (For example; "133111" $\in$ L but "313" $\notin$ L). Draw the transition diagram of the DFA for L. Explain the meaning of the states.

4)

(a) State the Halting Problem.

Using the undecidability of the halting problem and reduction, show that there is no algorithm to decide whether an arbitrary Turing machine halts when run with the input 111.

(b) Consider the Turing machine M in Fig. 3. Work out the binary representation R(M) using the technique given in the lectures. Using this R(M), give an example of a string that is a member of L_H and a string that is not a member of L_H, where $L_H = \{R(M)w : M \text{ halts with input w}\}$.

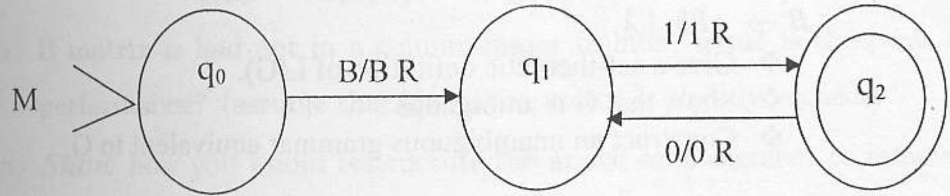

Fig. 3

(c) Use the Rice's theorem to show that the following property of recursively enumerable languages is undecidable: "L is context – free". (Note that you may use any result that you have proved/established in some other questions.)

5)

(a) Give a state diagram of a Turing machine that accepts the language **Palindrome** consisting of all strings over $\{a, b\}$ that is spelled the same forwards and backwards.

(b) Draw a state transition diagram for a Turing machine which computes the successor function on natural numbers in unary representation. Do not use macros.

(c) Using only the macros CPY_1 (copy), MR_1 (move right), ML_1 (move left) and A (add), define a sequential Turing machine which computes the function $f(n) = 3n$. That is, if the input is $B\bar{n}B$, the output will be $\underline{B}(\overline{3 * n})B$.

6)

(a) Design a context-free grammar for the Language
$$L = \{a^i b^j c^k d^l \mid i + j = k + l\}.$$

(b) Convert the following context-free grammar to Chomsky normal form:

$$S \rightarrow aABC \mid a$$
$$A \rightarrow aA \mid a$$
$$B \rightarrow bcB \mid bc$$
$$C \rightarrow cC \mid c.$$

(c) Let G be the grammar

$$S \rightarrow aSb \mid aAb$$
$$A \rightarrow cAd \mid B$$
$$B \rightarrow aBb \mid \lambda.$$

- Give a set-theoretic definition of L(G).
- Show that G is ambiguous
- Construct an unambiguous grammar equivalent to G.