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EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2007/2008

SECOND SEMESTER (Aug./Sept.,2009)

MT 104 - DIFFERENTIAL EQUATIONS & FOURIER SERIES

Proper & Repeat

Answer all questions

Time : Three hours

1. (a) State the necessary and sufficient condition for the differential equation

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

to be exact.

Hence solve the following differential equation

$$(y^2 - x^2 \sin xy)\frac{dy}{dx} = xy \sin xy - \cos xy - e^{2x}.$$

(b) Show that the solution of the general homogeneous equation of the first order and degree $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ is

$$\log x = \int \frac{dv}{f(v) - v} + C,$$

where $v = \frac{y}{x}$ and C is a constant.

Hence solve the differential equation

$$(x^2 - y^2)dx + 2xy\,dy = 0.$$

2. (a) If $F(D) = \sum_{i=0}^{n} p_i D^i$, where $D = \frac{d}{dx}$ and p_i, $i = 1, 2, ..., n$, are constants
$p_0 \neq 0$, Prove the following formulas:

i. $\frac{1}{F(D)} e^{\alpha x} = \frac{1}{F(\alpha)} e^{\alpha x}$, where α is a constant and $F(\alpha) \neq 0$;

ii. $\frac{1}{F(D)} e^{\alpha x} V = e^{\alpha x} \frac{1}{F(D+\alpha)} V$, where V is a function of x.

(b) Find the general solution of the following differential equations by using the re
in (a).

i. $(D^3 - 3D - 2)y = 540\, x^3 e^{-x}$.

ii. $(D^3 - D)y = e^x + e^{-x}$.

3. (a) Let $(1 + 2x) = e^t$. Show that

$$(1+2x)\frac{d}{dx} \equiv 2\mathcal{D},$$

and

$$(1+2x)^2 \frac{d^2}{dx^2} \equiv 4(\mathcal{D}^2 - \mathcal{D}).$$

where $\mathcal{D} \equiv \frac{d}{dt}$.

Use the above results to find the general solution of the following differential equa

$$[(1+2x)^2 D^2 - 6(1+2x)D + 16]y = 8(1+2x)^2.$$

(b) Solve the following simultaneous differential equations:

$$(D - 17)y + (2D - 8)z = 0,$$
$$(13D - 53)y - 2z = 0.$$

4. Use the method of Frobenius to obtain two linearly independent solutions in serie
the following differential equation

$$9x^2 y'' + 9x^2 y' + 2y = 0.$$

5. (a) Write down the condition of integrability of the total differential equation

$$P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0.$$

Hence solve the following equation

$$2(y+z)dx - (x+z)dy + (2y - x + z)dz = 0.$$

(b) Solve the following system of differential equations:

i. $\dfrac{dx}{y - zx} = \dfrac{dy}{yz + x} = \dfrac{dz}{x^2 + y^2}$;

ii. $\dfrac{dx}{2x} = \dfrac{dy}{-y} = \dfrac{dz}{4xy^2 - 2z}$.

(c) Find the general solution of the following linear partial differential equations:

i. $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$;

ii. $z = px + qy + \sqrt{1 + p^2 + q^2}$.

(d) Apply Charpit's method or otherwise to find the complete and the singular solution of the following non-linear first-order partial differential equation

$$pxy + pq + qy = yz$$

Here, $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$.

6. (a) Obtain Fourier series expansion of

$$f(x) = \begin{cases} 2x & \text{when} \quad 0 \leqslant x < 3, \\ 0 & \text{when} \quad -3 < x < 0 \end{cases}$$

(b) Use the finite Fourier transformation to show the solution of the partial differential equation

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2},$$

subject to the boundary condition:

$V(0,t) = 0, \quad V(4,t) = 0, \quad V(x,0) = 2x$, where $0 < x < 4, \ t > 0$

$$V(x,t) = \frac{-16}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} e^{\frac{-n^2\pi^2 t}{16}} \cos n\pi \sin \frac{n\pi x}{4}.$$