



EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2007/2008

SECOND SEMESTER (Aug./Sept., 2009)

MT 104 - DIFFERENTIAL EQUATIONS & FOURIER SERIES

Proper & Repeat

Answer all questions

Time : Three hours

1. (a) State the necessary and sufficient condition for the differential equation

$$M(x, y) dx + N(x, y) dy = 0$$

to be exact.

Hence solve the following differential equation

$$(y^2 - x^2 \sin xy) \frac{dy}{dx} = xy \sin xy - \cos xy - e^{2x}.$$

- (b) Show that the solution of the general homogeneous equation of the first order and degree $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ is

$$\log x = \int \frac{dv}{f(v) - v} + C,$$

where $v = \frac{y}{x}$ and C is a constant.

Hence solve the differential equation

$$(x^2 - y^2)dx + 2xy dy = 0.$$

2. (a) If $F(D) = \sum_{i=0}^n p_i D^i$, where $D = \frac{d}{dx}$ and $p_i, i = 1, 2, \dots, n$, are constants $p_0 \neq 0$, Prove the following formulas:

i. $\frac{1}{F(D)} e^{\alpha x} = \frac{1}{F(\alpha)} e^{\alpha x}$, where α is a constant and $F(\alpha) \neq 0$;

ii. $\frac{1}{F(D)} e^{\alpha x} V = e^{\alpha x} \frac{1}{F(D + \alpha)} V$, where V is a function of x .

- (b) Find the general solution of the following differential equations by using the results in (a).

i. $(D^3 - 3D - 2)y = 540x^3 e^{-x}$.

ii. $(D^3 - D)y = e^x + e^{-x}$.

3. (a) Let $(1 + 2x) = e^t$. Show that

$$(1 + 2x) \frac{d}{dx} \equiv 2D,$$

and

$$(1 + 2x)^2 \frac{d^2}{dx^2} \equiv 4(D^2 - D).$$

where $\mathcal{D} \equiv \frac{d}{dt}$.

Use the above results to find the general solution of the following differential equation

$$[(1 + 2x)^2 D^2 - 6(1 + 2x)D + 16]y = 8(1 + 2x)^2.$$

- (b) Solve the following simultaneous differential equations:

$$(D - 17)y + (2D - 8)z = 0,$$

$$(13D - 53)y - 2z = 0.$$

4. Use the method of Frobenius to obtain two linearly independent solutions in series form for the following differential equation

$$9x^2 y'' + 9x^2 y' + 2y = 0.$$

5. (a) Write down the condition of integrability of the total differential equation

$$P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0.$$

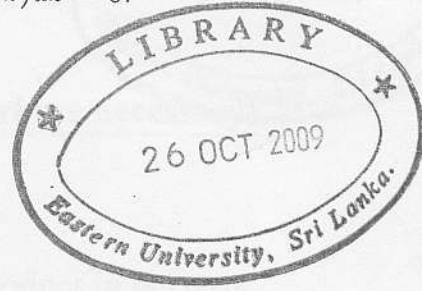
Hence solve the following equation

$$2(y + z)dx - (x + z)dy + (2y - x + z)dz = 0.$$

- (b) Solve the following system of differential equations:

i. $\frac{dx}{y - zx} = \frac{dy}{yz + x} = \frac{dz}{x^2 + y^2};$

ii. $\frac{dx}{2x} = \frac{dy}{-y} = \frac{dz}{4xy^2 - 2z}.$



- (c) Find the general solution of the following linear partial differential equations:

i. $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx;$

ii. $z = px + qy + \sqrt{1 + p^2 + q^2}.$

- (d) Apply Charpit's method or otherwise to find the complete and the singular solution of the following non-linear first-order partial differential equation

$$pxy + pq + qy = yz$$

Here, $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}.$

6. (a) Obtain Fourier series expansion of

$$f(x) = \begin{cases} 2x & \text{when } 0 \leq x < 3, \\ 0 & \text{when } -3 < x < 0 \end{cases}$$

- (b) Use the finite Fourier transformation to show the solution of the partial differential equation

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2},$$

subject to the boundary condition:

$$V(0, t) = 0, \quad V(4, t) = 0, \quad V(x, 0) = 2x, \quad \text{where } 0 < x < 4, \quad t > 0$$

$$V(x, t) = \frac{-16}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} e^{\frac{-n^2\pi^2 t}{16}} \cos n\pi \sin \frac{n\pi x}{4}.$$