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EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE -2007/2008

SECOND SEMESTER (Aug/Sept., 2009)

MT 105 - THEORY OF SERIES

(PROPER/REPEAT)

Answer all Questions **Time: One hour**

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1. (a) Define what is meant by the convergent or divergent of an infinite series $\sum_{n=1}^{\infty} a_n$.

   Show that the series

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \frac{1}{7\cdot 9} + \cdots,$$

   is convergent and find its sum.

   (b) Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series of real numbers.

   i. Show that if $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$ as $n \to \infty$.

   ii. Is it true that, if $a_n \to 0$ as $n \to \infty$ then the series $\sum_{n=1}^{\infty} a_n$ converges? Justify your answer.

2. (a) Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be series of positive real numbers such that $\left(\frac{a_n}{b_n}\right)$ tends to a finite non-zero limit as $n \to \infty$. Prove that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge.

(b) Determine whether the following series converge or diverge:

i. $2+\dfrac{3}{2^3}+\dfrac{4}{3^3}+\dfrac{5}{4^3}+\cdots,$

ii. $1+\dfrac{2^2+1}{2^3+1}+\dfrac{3^2+1}{3^3+1}+\dfrac{4^2+1}{4^3+1}+\cdots.$

(c) i. Let $(a_n)_{n=1}^{\infty}$ be a decreasing sequence of positive terms such that $a_n \to 0$ as $n\to\infty$. Show that the series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ converges.

ii. Prove that $\sum_{n=1}^{\infty}(-1)^{n+1}\sin\left(\dfrac{1}{n}\right)$ converges. What will happen to this seri

if we drop the factor $(-1)^{n+1}$? Justify your answer.