21 2004
Eastern University, Sri Lanka

EASTERN UNIVERSITY, SRI LANKA

SPECIAL DEGREE EXAMINATION IN MATHEMATICS-2001/2002

Part II(JAN./ FEB. 2004)

MT 412 – DATA STRUCTURES & DESIGN OF ALGORITHMS

Answer Four Questions Only

Time : Three hours

Q1

Describe the following Algorithmic Techniques:
(a) Divide-and-Conquer algorithms
(b) Greedy algorithms

Give suitable example in each case.

(i) A large problem of size ***n*** can be divided into ***a*** sub problems each of size ***n/c.*** the amount of time required to perform this decomposition as well as the time required to combine the solutions to the sub problems in order to produce the final solution is ***b*n.*** Let T(n) be the running time for the problem of size ***n*** and assume T(1) is equal to ***b***.

Prove that,

$$
\begin{aligned}
\textit{for } n>1 \qquad T(n) &= O(n) \quad \textit{if} \quad a<c \\
&= O(n\log_c n) \quad \textit{if} \quad a=c \\
&= O(n^{\log_c a}) \quad \textit{if} \quad a>c
\end{aligned}
$$

where ***a, b,*** and ***c*** are constants

Describe a divide-and-conquer algorithm to multiply two n-bit integers and obtain the time complexity of your method.

(ii) A single server is set up to service a sequence of customers. Assume that we know in advance how much time is required to service each customer. Use a Greedy algorithm to minimize the average time spent on each customer.

Q2

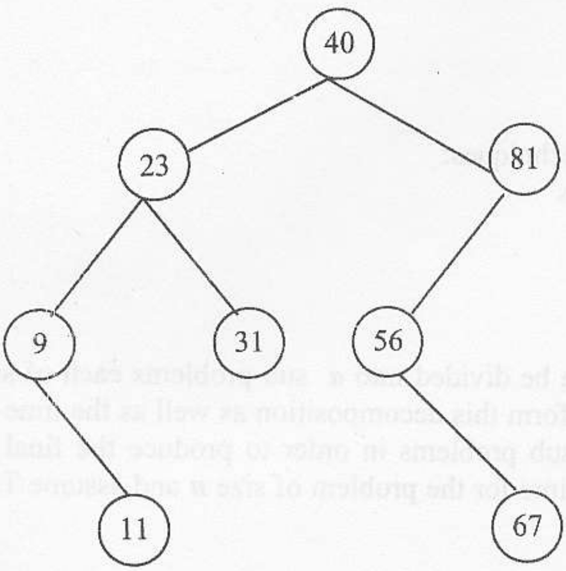
(a) Obtain the worst case, best case and average case running time of the following sorting algorithms:

(i) Bubble sort
(ii) Insertion sort

(b) Describe the Quicksort algorithm and obtain the worst case, best case and average running time of Quicksort algorithm.

Q3

(a) Consider the following Binary Search Tree (BST):



Illustrate how the following are inserted in that order in the above BST:
27, 88, 5, 27

Describe a step by step sequence for carrying out the following deletions in that order from the *original* BST (the one shown in the diagram):
81, 40

(b) Describe the following terms in detail
- Open hashing and Closed hashing
- Linear probing

A Hash table is required to store a collection of ***distinct integers***. The required operations for this Hash table only ***inserting*** a new element, ***deleting*** an element and ***searching*** for element. Describe a ***closed hash strategy*** with N-buckets to create the hash table. T hashing function to be used is $h(i) = i \bmod N$, with linear probing strategy for collision resolution.

Give a step by step illustration for the effect of performing the following operations on an initially empty hash table with bucket size 11. For each operation the sequence of intermediate slots visited should be shown explicitly.

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Insert	35, 52, 67, 98, 45, 29, 18, 72
Search	56, 45
Delete	67, 29, 45
Insert	56, 18

In the case of closed hashing derive a formula for the cost of insertion if the alternative locations used by the rehashing strategy are chosen at random.

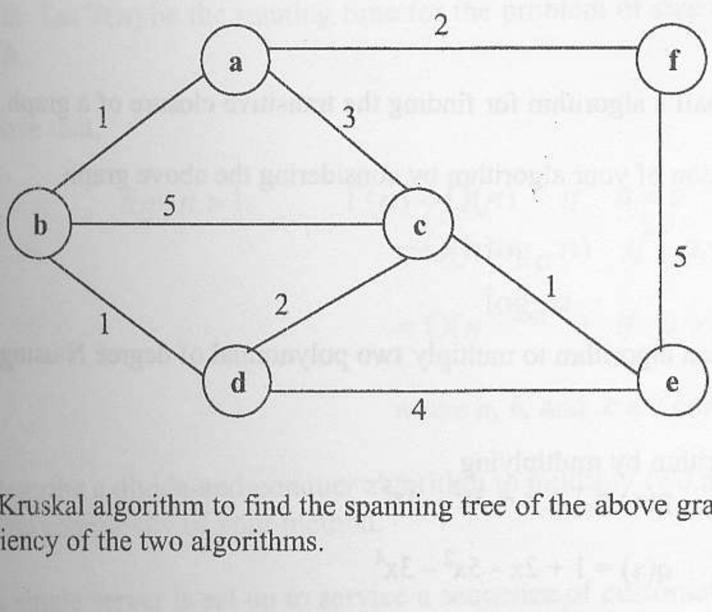
Q4

(a) Describe the ***adjacency list*** representation of a graph G = (V, E).

(b) Define a ***spanning tree*** of an undirected graph G.
Illustrate your definition by an example.

(c) Describes Prim's spanning tree algorithm and show that the complexity of Prim's spanning tree algorithm is $O(n^2)$.

(d) Find the spanning tree of the following graph:



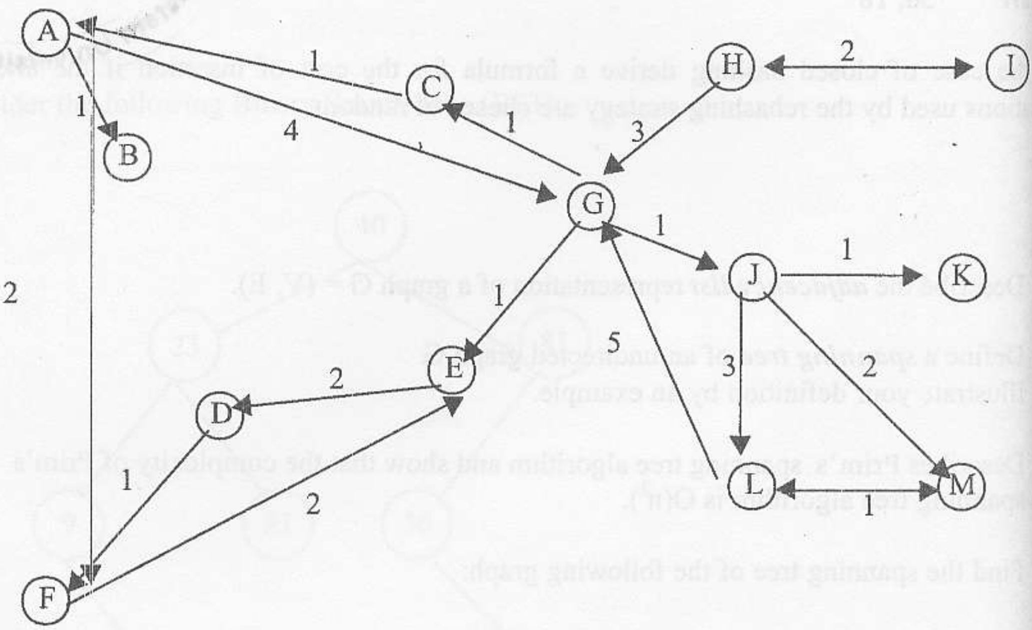
(e) Use Kruskal algorithm to find the spanning tree of the above graph and discuss the efficiency of the two algorithms.

Q5

(a) Describe the Dijkstra's single-source shortest paths algorithm.

(b) Describe the Floyd's all-pairs shortest paths algorithm and show that the complexity the algorithm is $O(n^3)$.

(c) Solve the all-pairs shortest paths problem for the following directed weighted graph Floyd's algorithm:



(d) Describe the Warshall's algorithm for finding the transitive closure of a graph.

Illustrate the operation of your algorithm by considering the above graph.

Q6

(a) Describe, in detail, an algorithm to multiply two polynomial of degree N using $N^{1.58}$ multiplications.

Illustrate your algorithm by multiplying

$$p(x) = 1 + x + 3x^2 - 4x^3$$

$$q(x) = 1 + 2x - 5x^2 - 3x^4$$

(b) Describe an interpolation based algorithm to multiply two polynomials of degree N

Illustrate your algorithm for N=8.