LIBRARY
0 2 JUN 2010
Eastern University, Sri Lanka.

EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
SECOND EXAMINATION IN SCIENCE - 2008/2009
FIRST SEMESTER(Feb./Mar., 2010)
MT 207 - NUMERICAL ANALYSIS
(PROPER & REPEAT)

Answer all Questions **Time: Two hours**

---

1. (a) For a five digit decimal floating point number, define the terms **mantissa** and **characteristic.** [20 marks]

   (b) Explain what is meant by **underflow** and **overflow** in a calculation. [15 marks]

   (c) Describe the truncation error when $\sin x$ is approximated by x, where

   $$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots .$$

   [20 marks]

   (d) Suppose $a = 0.471 \times 10^{-2}$ and $b = -0.185 \times 10^{-4}$, use 3 digit floating point arithmetic to compute $a+b$ and $a-b$ and find the rounding error in each case. Hence find the rounding error in calculating $a^2 - b^2$. [25 marks]

   (e) For a binary computer working with 4 digit in the mantissa, floating point arithmetic, find the bound of the relative round-off error. [20 marks]

2. (a) Using bisection method find a root of the equation $t^3 + t - 9 = 0$ correct to o decimal place. [25 mark

(b) Define fixed points of the iteration

$$s_{n+1} = s_n + \lambda(s_n^3 + s - 9).$$

Find an approximate range of values of the constant λ such that the iterati converges to the fixed point near 2. [20 mark

(c) Find the order of a given method described by an equation

$$\epsilon_{r+1} - \epsilon_r = -\frac{2\epsilon_r f'(\alpha) + \epsilon_r^2 f''(\alpha)}{2[f'(\alpha) + \epsilon_r f''(\alpha)]},$$

where ϵ_r is the error in approximation after the r^{th} iteration. [25 mark

(d) A calculator is defective: it can only add, subtract and multiply. Using th defective calculator and the Newton-Raphson method find an iterative formu to find $\frac{1}{a}$, where a is real number. Hence find a sequence of approximate valu to $\frac{1}{1.37}$ correct to 4 decimal places. [30 mark

3. (a) Complete the finite difference table for the function $f(x)$ given by

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$
−1.0	0.130					
		0.016				
−0.5	0.146					
		0.023				
0.0	0.169					
		0.033				
0.5	0.202					
		0.047				
1.0	0.249					
		0.065				
1.5	0.314					

LIBRARY
02 JUN 2010
Eastern University, Sri Lanka.

Use a Newton's forward formula to find an approximation to $f(-0.3)$ correct to 3 decimal places. [35 marks]

(b) Using the table given in (a) and a quadratic Lagrange's interpolation, find an approximation z to $f'(-0.3)$. Further, using the error term in Lagrange interpolation find an expression for $z - f'(x)$ by assuming that the third derivative of $f(x)$ is constant in the interval you have chosen. [40 marks]

(c) The error in Simpson's rule for approximating the integral $I = \int_a^b f(x)\,dx$ is

$$-\frac{(b-a)^5}{2880} f^{(4)}(\xi), \quad a < \xi < b.$$

Show that the error in the Composite Simpson's rule for integrating $f(x)$ from a to b is

$$-\frac{(b-a)}{180} h^4 f^{(4)}(\theta), \quad a < \theta < b.$$

(Assume that the fourth derivative of $f(x)$ is continuous in $[a, b]$)

[25 marks]

4. (a) Estimate

$$\int_0^1 e^{x^2} dx$$

using Simpson's rule and Composite Simpson's rule with 2 strips. If the ex value of the above integral is 1.46265, what is the rate of decrease of the er with respect to the strip width? [35 ma

(b) Solve the system of equations:

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 0 \\ -x_1 + 4x_2 + x_3 &= 3 \\ 2x_1 + x_2 + 5x_3 &= 1 \end{aligned}$$

by Gaussian elimination and back substitution without interchanging the eq tions. [25 mar

(c) Carry out 2-iterations of the Jacobi method for solving

$$\begin{aligned} 10x - y - 3z &= 1 \\ x + 10y - 2z &= 0 \\ 3x + 2y + 10z &= -2 \end{aligned}$$

with initial estimation $(x, y, z)^T = (0, 0, 0)^T$.

Describe briefly how the Gauss-Seidel method differs from the Jacobi meth according to the above problem.

[40 marl