EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD EXAMINATION IN SCIENCE – 2008/2009

FIRST SEMESTER (Feb., 2010)

ST 303- REGRESSION ANALYSIS AND QUALITY CONTROL

NSWER ALL QUESTIONS TIME: THREE HOURS

STATISTICAL TABLES SHOULD BE PROVIDED

a) Explain what is meant by "process capability"? Why it is important? What does it tell us? Give two methods used to measure process capability? [20 marks]

b) Samples n=5 units are taken from a process every hour. The $\bar{X}$ and R values are determined. After 25 samples have been collected we calculate $\bar{\bar{X}}$=20 and $\bar{R}$=4.56.

i. What are the 3-sigma control limits and center line for $\bar{X}$ and R charts? [30 marks]

ii. Both charts exhibit control. Estimate the process standard deviation? [10 marks]

iii. If the process mean shifts to 24, what is the probability of not detecting this shift on the first subsequence sample? [15 marks]

iv. Assume that the process output is normally distributed. If the specifications are 19±5, what is your conclusion regarding the process capability? [15 marks]

v. What is the fraction non-conforming items produced by the process? [10 marks]

2.

a) Describe the process of *"Acceptance Sampling"*? What types of sampling plans are there? Briefly explain them? What is acceptance sampling used for? [25 marks]

b) Stating any assumptions, construct an OC curve for a sampling plan in which a sample of n=5 items are drawn from a lot of N=1000 items. The accept/reject criteria are set up in such a way that we accept a lot if no more than one defect is found. [25 marks]

Hint: Cumulative probability of a binomial distribution is presented below :

		Proportion of items defective (p)									
		0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
n	x										
	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.031
	1	0.9974	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.187
	2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.500

; Where n=sample size and "x" column tells us the cumulative number of defects fou

at which we reject the lot.

c) What is meant by *"Average "Outgoing Quality, (AOQ)"* ? Construct an AOQ Curve for above sampling plan and interpret the meaning of the curve. [20 ma

d) What is the *"Average Outgoing Quality Limit"* (AOQL)? [5 ma

e) What is *"Average Total Inspection"* (ATI)? [10 ma

f) Plot a ATI versus *"Incoming Lot Quality"* curve. [15 ma

3.

a) Discuss the concepts of chance and assignable cause of variability and the part they in Statistical Process Control. [20 ma

b) What is meant by the statement that a process is in a state of control? [10 ma

c) Is the control chart equivalent to a statistical test of a hypothesis? [10 ma

d) Describe how you would construct and interpret the following charts in statistical control [10 m

i. P-Chart [10 m

ii. C-Chart

The following are the number of imperfections per yard of a yarn.

5 3 4 8 2 3 1 2 5 9 2 2 2 3 4

Is there evidence that the process is out of control? Find the control limits for the pr [40 m

4.

a. Using *Ordinary Least Squares* criterion, derive estimated coefficients for the true sl intercept of a simple linear regression model. [30

b. Using matrix notations, fit a simple linear regression model t o the following data

X	Y
1	8
2	17
3	29
4	34
5	46
6	42
7	52

You may assume $(X^T X)^{-1} = \begin{pmatrix} 20/28 & -4/28 \\ -4/28 & 1/28 \end{pmatrix}$ and $(X^T Y) = \begin{pmatrix} 228 \\ 1111 \end{pmatrix}$

Find the estimated value of Y at X=4.0 and the standard error of the estimate. [70 marks]

The following table gives the experience (in years) and the number of computers sold during the previous three months by seven sales persons.

Experience	4	12	9	6	10	16	7
Computers sold	19	42	28	33	39	35	23

a. Draw a scatter diagram for these data, taking number of computers sold as a response variable and experience as a predictor. [05 marks]

b. Write a statistical model of a simple linear regression. [05 marks]

c. Find the least squares estimates of the slope and intercept. Give a brief interpretation of the estimated regression coefficients. [15 marks]

d. Compute r^2 and explain what it means. [20 marks]

e. Predict the number of computers sold during the past three months by a sales person with one year of experience, find the error for this estimate and give an appropriate warning with regard to the prediction. [30 marks]

f. Construct 95% confidence interval for the slope. [15 marks]

g. Testing at 2.5% significance level, can you conclude that the slope is greater than zero. [10 marks]

6.

Consider the data in the following table:

X_0	X_1	X_2	Y
1	1	8	6
1	4	2	8
1	9	-8	1
1	11	-10	0
1	3	6	5
1	8	-6	3
1	5	0	2
1	10	-12	-4
1	2	4	10
1	7	-2	-3
1	6	-4	5

a. Write a model specification matrix for a model of the form,

$Y = \beta_0 X_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$ [05 ma

b. Estimate the β's in the above model. [10 ma

c. Write out the Analysis of Variance table. [30 ma

d. Using α = 0.05, test to determine if the over all regression model is statis significant. [20 m

e. What proportion of the total variation about $\bar{Y}$ is explained by the two variables [10 m

f. The inverse of the $(X^T X)$ matrix for this problem is :

$$\begin{bmatrix} 4.3705 & -0.8495 & -0.4086 \\ -0.8495 & 0.1690 & 0.0822 \\ -0.4086 & 0.0822 & 0.0422 \end{bmatrix}$$

Calculate the estimates of the following:

1. Variance of estimated β_1.
2. Variance of estimated β_2 .
3. The variance of predicted value of Y for the point $X_1 = 3$ and $X_2 = 5$.

[25