EASTERN UNIVERSITY, SRI LANKA

FIRST EXAMINATION IN SCIENCE (2001/2002)

FIRST SEMESTER

(April/May ' 2002)

MT 106 - TENSOR CALCULUS

Answer all Questions **Time : One hour**

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1. (a) Define the terms **symmetric** and **skew-symmetric** tensors.

   i. If A^{pq} and B_{rs} are skew-symmetric tensors, show that $C^{pq}_{rs} = A^{pq}\, B_{rs}$ is symmetric.

   ii. If $ds^2 = g_{ij}\, dx^i\, dx^j$ is an invariant, show that g_{ij} is a symmetric covariant tensor of rank two.

   (b) Evaluate the following:

   $\delta^p_q\, \delta^r_s\, A^{qs}$, $\qquad\qquad$ $\delta^p_q\, \delta^q_r\, \delta^r_s\, \delta^s_p$.

   (c) Find the covariant and contravariant components of a tensor in cylindrical coordinates $(\rho,\ \theta,\ z)$ if its contravariant components in rectangular coordinates $(x,\ y,\ z)$ are $xy,\ \ 2y - z^2,\ \ xz$.

2. (a) Define the following terms:

   i. **Christoffel symbols** of first and second kind;

   ii. The **covariant derivative** as applied to a tensor of type A^{ij}_{k}.

   (b) Determine the christoffel symbols of second kind corresponding to the metric ds, given by

$$ds^2 = (dx^1)^2 + [(x^2)^2 - (x^1)^2](dx^2)^2$$

   and also find the Geodesic equations.

   (c) Prove that $A_{p\,;qr} - A_{p\,;rq} = R^{n}_{pqr}A_n$ where A_n is an arbitrary covariant tensor of rank one.