



EASTERN UNIVERSITY, SRI LANKA

FIRST EXAMINATION IN SCIENCE 2002/2003

(June./July, '2003)

FIRST SEMESTER

MT 106 - TENSOR CALCULUS

Answer all questions

Time : One hour

1. (a) Define the following terms:

i. Covariant tensor

ii. Contravariant tensor.

(b) Write the transformation equations for the following tensors

$$A_k^{ij}, B_{jkl}^m, C^{mn}.$$

(c) Show that if B_q is an arbitrary covariant tensor of rank one and C^p is a contravariant tensor of rank one, then $A(p, q)$ must be a tensor of rank two.

(d) A covariant tensor has components $xy, 2y - z^2, xz$ in rectangular co-ordinates (x, y, z) . Find its contravariant component in cylindrical polar co-ordinates (P, θ, z) .

2. (a) Define the following:

- i. Christoffel symbols of first and second kind
- ii. Geodesics

Explain the term "covariant derivative" as applied to a tensor of type A_r^p .

(b) With usual notations, prove the following:

i. $[pq, r] = [qp, r]$.

ii.
$$\frac{\partial g^{rk}}{\partial x^m} = -g^{ir}\Gamma_{im}^k - g^{jk}\Gamma_{jm}^r.$$

Hence deduce that the covariant derivative of metric tensors g^{rk} and g_{rk} are zero.

(c) Determine the Christoffel symbols of second kind for the line element

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

and find the corresponding geodesic equations.