EASTERN UNIVERSITY, SRILANKA

FIRST EXAMINATION IN SCIENCE (2002/2003)

FIRST SEMESTER

(June/July '2003)

ST 101-PROBABILITY THEORY

Answer all Questions **Time allowed: Three hours**

Qu1. a) Define the term "conditional probability".

There are two children in a family. Assume that the chances of being a boy or girl in the family are equal. What is the probability that both are boys, given that at least one is boy?

b) Prove the following laws, assuming that in each case, the conditional probabilities are defined.

i) If $P(B)=1$ then $P(A/B)= P(A\cap B)$.

ii) If A and B are disjoint, and $P(B)>0$ then $P(A/B) =0$.

iii) If $A_1 \subseteq A_2$ then $P(A_1/B) \leq P(A_2/B)$.

iv) $P(A_1 \cup A_2 /B) = P(A_1/B)+P(A_2/B)-P(A_1\cap A_2/B)$.

c) For any two events A and B, Verify that the following statements are true or not.

$P(A/B)+P(A^c/B)=1.$

$P(A/B)+P(A/B^c)=1.$

Qu2.a) State the "Bayes" theorem for conditional probability.

There are two modes of transportation from Colombo to a ci the south, namely, by land or sea. Land transportation ma rail or highway. About 70% of the material is transport land and the rest is transported by sea. Also 60% o transportation is by highway and the rest by rail. percentage of damaged cargo are 10% by highway, 6% by and 5% by sea.

i) What percentage of the total cargo may be expected to damaged?

ii) If a damaged cargo is received, what is the probabilit that it was shipped by, I) Sea? II) Rail? III) Land?

b) The failure of the circuit board causes a computing syst be shut down until a new board is delivered. Delivery ti is uniformly distributed over the interval 1 to 5 days total cost C of this failure and shutdown consists of a cost C_0 and replacement cost of C_1 which is proportion X^2, so that,

$$C = C_0 + C_1X^2 .$$

i) Find the probability that the delivery time is 2 or mo days.

ii) Find the total expected cost of a single computer if t fixed cost is Rs.500 and the replacement cost is Rs.25 day.

iii) Calculate the standard deviation of the above total co

Qu3. a) Define what is meant by "Random variable".

Let X be a continuous random variable and let a and b constants. Show that,

$E(aX+b) = a\,E(X) + b$ and $Var(aX+b) = a^2 Var(X)$.

b) Let X be a continuous random variable with probability density function,

$$f_X(x)=\begin{cases}0 & ; x\le 0\\ kx^3 & ; 0<x<1\\ ke^{1-x} & ; x\ge 1\end{cases}$$

i) Evaluate k.

ii) Calculate $P(0.5<X<2)$ and $P(X>2/X>1)$.

iii) Find the mean and variance of X and mean and variance of $Y=2X+7$.

Qu4. Define the probability distributions for discrete and continuous random variables.

a) Prove the following properties of the expected value.

i) $E(c)=c$; where c is a constant.

ii) $E(cg(X))= cE(g(X))$.

iii) $E[c_1g_1(X)+c_2g_2(X)]=c_1E(g_1(X))+c_2E(g_2(X))$.

iv) $E(g_1(X))\le E(g_2(X))$; if $g_1(x)\le g_2(x)$ *for* all x.

b) In a game, an unbiased die is thrown and a banker pays money (Rs) twice the number on the side of the die, which is uppermost. What would be the expected value that the banker pays out?

c) A fair coin is tossed twice. Let R be the number of heads appears in the toss. Find the distribution of $1/(R+1)$ and hence calculate it's expectation. Show that,

$$E[1/(R+1)]\ne 1/[E(R)+1].$$

Qu5 a) An archer shoots arrows at a target. The distance, X cm, from the center of the target at which an arrow strikes the target has probability density function, f, defined by,

$$f(x)=\frac{1}{10}e^{-\frac{x}{10}}\ ;\ \text{Where } x\ge 0.$$

1. Name the above probability distribution.

2. Find the distribution function, $F(x)$.

3. An arrow scores eight points if $X \leq 2$, five points $2<X\leq 5$, one point if $5<X\leq 15$ and no points other wi

Find the expected score when an arrow is shot at target.

b) A manufacturer buys silicon chips in lots of 25. In or to check the quality of each lot he puts a rand selected sample of size 4 from the each lot to test. Le be the number of defective chips in the sample. following criterion will be used: If $X\leq 1$, accept the other wise reject the lot. Find the probability that a containing 5 defective chips will be accepted.

c) The amount of distilled water dispensed by a cer machine is normally distributed with mean value 64 oz standard deviation 0.78 oz. What container size c ensure that overflow occurs 0.5% of the time?

a) Define "moment generating function" of a random variable.

b) I) Let X is a binomial random variable with parameters and p.

A) Show that $M_X(t) = (q + pe^t)^n$.

B) Hence derive that the mean$=\mu_X$ and variance$=\sigma_X^2$ of

C) For what value of p, Var(X) is maximized if n is fixed?

D) If n=25 and p=0.2, evaluate $P(X<\mu_X-\delta_X)$.

II) In testing lethal concentration of a chemical found in polluted water it is found that a certain concentration will kill 20% of the fish that are subjected to it for 24 hours. If 20 fish are placed in a tank containing this concentration of chemical, find the probability that after 24 hours,

i) Exactly 14 survive.

ii) At least 10 survive.

iii) At most 16 survive.

c) In a particular department store customers arrive at a check point counter according to Poisson distribution at an average of 7 per hour. During a given hour, what are the probabilities that,

a) No more than three customers arrive.

b) At least two customers arrive.

c) Exactly five customers arrive.