EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 2002/2003

(June./July.'2003)

FIRST SEMESTER

MT 207 - NUMERICAL ANALYSIS

Answer all questions **Time: Two hours**

1. Define absolute error and relative error of a numerical value.

   (a) Suppose x_1 and x_2 are approximations to the true values α and β respectively and e_1 is the error due to round-off α to k_1 decimal places and e_2 is the error due to round-off β to k_2 decimal places. Find a bound on the absolute error in $x_1 + x_2$ and $x_1 - x_2$.

   i. The numbers a and b when rounded to 3 decimal places are 3.724 and 2.251 respectively. Find $b - a$ to two decimal places and show that the third decimal figure is 2, 3 or 4.

   ii. The numbers c and d when rounding to 4 significant digits are 23.86 and 0.01762 respectively. Show that the true value of $c - d$ to 4 significant figures will be either 23.85 or 23.84.

   (b) Evaluate the roots of the quadratic equation

   $$x^2 - 60x + 1 = 0$$

   using 4 significant digits throughout the calculation. Obtain the relative error in the roots and discuss.

2. Define the order of convergence of an iterative method to compute the roots of a nonlinear equation

$$f(x) = 0 \text{ ------------ } (1)$$

(a) Obtain Newton-Raphson algorithm to compute the roots of the equation (1) in an interval $[a, b]$.

Show that the order of convergence of Newton-Raphson algorithm is at least 2.

(b) Obtain Secant method to compute the root of the equation (1) in an interval $[a, b]$.

Show that the order of convergence of Secant method is approximately 1.62.

Compute the root of the equation

$$f(x) = 3x + \sin x - e^x$$

near $x = 0$ using the methods (a) and (b) to 3 decimal places accuracy and discuss the efficiency of (a) and (b).

3. Suppose $x_0, x_1, ..., x_n$ are distinct numbers in the interval $[a, b]$ and $f \in C^{n+1}[a, b]$. Obtain a unique polynomial $p_n(x)$ of degree at most n with the property

$$f(x_k) = p_n(x_k) \quad \text{for each } k = 0, 1, .., n$$

and show that

$$f(x) - p_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)(x - x_1)...(x - x_n)$$

for each x in $[a, b]$, where $\xi(x) \in (a, b)$.

Suppose a table is to be prepared for the function $f(x) = e^x, \quad 0 \leq x \leq 1$. Assume the number of places to be given per entry is $d \geq 6$ and that the difference between adjacent x-values, the step size is h.

Show that

$$| f(x) - p_1(x) | \leq \frac{eh^2}{8}$$

where $p_1(x)$ is a linear interpolation polynomial, and estimate h to give an absolute error of at most 10^{-6}.

4. (a) Obtain Composite Simpson's rule to estimate $\int_a^b f(x)dx$ and show that the truncation error is less than or equal to $\frac{1}{180}h^4(b-a) \mid f^{(iv)}(\xi) \mid$, where $\mid f^{(iv)}(\xi) \mid = \max_{a \leq x \leq b} \mid f^{(iv)}(x) \mid$.

   Estimate the truncation error in the value of $\int_2^4 (1+x)^{\frac{1}{2}} dx$ with $h = 0.5$.

   (b) Describe the Gaussian Elimination with scaled partial pivoting for the solution of the equation

$$A\underline{x} = \underline{b}$$

   with the usual notation.