EASTERN UNIVERSITY, SRI LANKA

FACULTY OF COMMERCE AND MANAGEMENT

DEPARTMENT OF COMMERCE

FIRST YEAR/ FIRST SEMESTER EXAMINATION IN BUSINESS ADMINISTRATION/ COMMERCE 2003/ 2004 (NOVEMBER/ DECEMBER 2004)

COM 1023 BUSINESS MATHEMATICS AND STATISTICS

Answer all Questions **Time : 03 Hours**

01. A. (I) Express the following in its simplest form

(i) $\left(x^3 y^{\frac{4}{3}}\right)^2 + \sqrt{x}\left(x^3 + y^{\frac{4}{3}}\right)$ (ii) $\left(x - \frac{2}{x+1}\right) \div \left[1 - \frac{(4x+7)}{(x^2+4x+3)}\right]$.

(II) Factorise the following expressions completely

(i) $3x^{2n} + 7x^n + 2$ (ii) $x^6 + 8y^6$.

(III) Solve the following equations

(i) $4^x = 8^{3-x}$ (ii) $\log(2x+1) + \log(x+3) = \log(12x+1)$.

(60 marks)

B. (I) The manufacturer of a certain product can sell all units at a price of Rs. 20 each. It costs him Rs. 12.50 to produce each item in materials and labour, and he has additional overhead costs of Rs. 7,000 per month in order to operate the plant. Find the number of units he should produce and sell to make a profit of Rs. 5,000 per month.

(15 marks)

(II) A firm manufactures two products P_1 and P_2. Each product has to be processed by two machines M_1 and M_2. Each unit of type P_1 requires 01 hour of processing by machine M_1 and 2.5 hours by machine M_2, and each unit of type P_2 requires 04 hours of machine M_1 and 01 on machine M_2.

Total available time for machine M1 is 400 hours and for machine M2 is 550 hours each month. How many units of each type can be manufactured in one month if the total available time on the two machines is to be utilized?

(25 marks)

02. A. Let L be the line passing through the points (-2, -3) and (5, 4).

(I) Find the slope of this line.

(II) Find the equation of this line.

(III) Find the y-intercept point and x-intercept point of this line.

(IV) Find the intersection point of this line with the line $x + 2y = 0$.

(50 marks)

B. Of the equations $p + 6q = 420$ and $p = 6q + 60$, one is a supply function of a product and the other is a demand function of that product, where p is the price of the product and q is the quantity of the product.

(I) Sketch the two functions on the same axes.

(II) Label the demand and supply functions on the graph and give reasons for your choice.

(III) Find the equilibrium price and the quantity.

(50 marks)

03. A. (I) Calculate the matrix $C(2A - 3B)$, where

$$A = \begin{pmatrix} 2 & -5 \\ -3 & 7 \end{pmatrix}, \qquad B = \begin{pmatrix} -1 & -3 \\ -4 & 2 \end{pmatrix} \text{ and } \qquad C = \begin{pmatrix} 2 & 3 \\ -1 & 5 \\ 7 & -2 \end{pmatrix}$$

(II) If x satisfies $\begin{vmatrix} x-1 & 5x-3 \\ 2 & x+1 \end{vmatrix} = 0$, find x.

(III) If A is a matrix which satisfies

$$A + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ -5 & 7 \end{pmatrix} \text{ find } A.$$

(40 marks)

B. (I) Find the inverse matrix of the matrix,

$$\begin{pmatrix} 3 & 5 & 2 \\ 2 & 2 & 4 \\ 4 & 3 & 4 \end{pmatrix}.$$

(II) A factory makes three different brands of chocolates. Plain chocolate requires 3 units of cocoa, 2 units of sugar and 4 units of milk for each box; dark chocolate needs 5 units of cocoa, 2 units of sugar and 3 units of milk for each box; and light chocolate uses 2 units of cocoa, 4 units of sugar and 4 units of milk in each box. The factory has stocks of raw materials comprising 4,000 units of cocoa, 3,000 units of sugar and 4,000 units of milk.

(a) If x boxes of plain chocolate, y boxes of dark chocolate and z boxes light chocolate are produced in order to use up all of the raw material held in stock, give three linear equations with the three unknowns x, y and z using the information given above.

(b) Use the matrix inversion found in (I) to solve the three linear equations.

(60 marks)

04. A. Differentiate the following functions with respect to x

(I) $y = \left(x^3 + 3\right)^5$

(II) $y = \dfrac{(1+x)(1-x)}{\left(x^2+1\right)}$

(III) $y = \log\left(\sqrt{x} + x\right)$

(IV) $y = x^2\left(e^x - 1\right).$

(40 marks)

B. (I) A company producing a single product finds that each item produced cost Rs. 12 and that it has to pay Rs. 300 in fixed costs each week. If q units are produced each week, write down a function expressing the weekly costs in terms of q.

(II) The company operates as a monopoly in a market where the price is given by the function of q.

$$p = 24 - \frac{1}{20}q.$$

Find the revenue function in terms of q, and hence find the company's profit function.

(III) Find the value of q that maximizes profits and explain how you can be sure that it is a maximum.

(IV) Find the maximum profit.

(V) Find the break-even quantity.

(VI) Suppose that the capacity of the company is limited to 80 units of production per week. It can finance a doubling of capacity by taking out a loan, but the repayments will increase weekly costs to Rs. 400. Decide whether such an expansion will increase profits, justifying your answer.

(60 marks)

05. The annual income data in thousands of rupees for the 30 male policyholders and the 30 female policyholders are shown in the table below.

Class interval	Frequency	
	Male	Female
15 – 25	8	7
25 – 35	10	11
35 – 45	6	5
45 – 55	3	2
55 – 65	1	2
65 – 75	0	1
75 – 85	0	2
85 – 95	2	0

(I) Constrict a histogram for the female policy holder salaries and using the same axes construct a frequency polygon for the male policy holder salaries.

(II) Comment on the differences in the distribution of the male and female salaries.

(III) Find the following

(a) Median salary for male policy holder;

(b) Mode salary for female policy holder;

(c) Coefficient of variation for male policy holder;

(d) Coefficient of variation for female policyholder.

(IV) From your calculations in (c) and (d) which group had relatively more variation?

(100 marks)