

Answer all Questions

Time: 03 Hours

01. A) i) Convert to radical form.

$$\left(a^{1/3}b^{1/6}\right)\left(a^{3/4}b^{-2/3}\right)$$

- ii) Rationalize the denominator and simplify.

$$\frac{\sqrt{c}}{\sqrt{c} + \sqrt{d}} - \frac{\sqrt{d}}{\sqrt{c} - \sqrt{d}}$$

- iii) Examine whether the following numbers are rational or irrational.

a) $(\sqrt{5} + 5)^2$

b) $(5 - \sqrt{5})(5 + \sqrt{5})$

- B) Simplify the following expressions to the lowest terms.

i)
$$\frac{2(a+b)^{-1} - 5(a-b)^{-1}}{4(a^2 - b^2)^{-1}}$$

ii)
$$\frac{3}{x+y} + \frac{2x^2 - 2xy + 4x - 4y}{4x+8} \div \frac{y^2 - x^2}{2y}$$

iii)
$$\frac{(2^{2n} - 3 \cdot 2^{2n-2})(3^n - 2 \cdot 3^{n-2})}{3^{n-4}(4^{n+3} - 2^{2n})}$$

iv)
$$\frac{\frac{5}{a+2} - \frac{1}{a-2}}{\frac{3}{2+a} + \frac{6}{2-a}}$$

- C) i) If
- $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{1}{2}$
- find the value of
- $\frac{a^2+ab+b^2}{a^2-ab+b^2}$

- ii) If
- $a^b = b^a$
- show that
- $\left(\frac{a}{b}\right)^{a/b} = a^{(a/b-1)}$

(20 Marks)

02. A) Solve the following inequalities for x . Write each solution using inequality notation and interval notation.

i) $x - 6 < 3x < 2x + 5$ ii) $x^2 + x - 6 \geq 0$

B) Solve the following equations for x .

i) $\frac{x}{2x^2 + 3x - 2} - \frac{1}{2x} = \frac{3}{x^2 + 2x}$

ii) $\sqrt[3]{x + 1} + 5 = 3$

iii) $x^{2/3} - 5x^{1/3} + 6 = 0$

iv) $x^2(3x - 2) = x$

C) Factor the following expressions completely.

i) $x^{4n} - 16$

ii) $x^2 - 4x + 4 - y^2$

iii) $x^6 + 6x^3 - 16$

iv) $36xy^2 - 48xyz^2 + 16xz^4$

(20 Marks)

03. A) The cost in rupees per day to operate a small delivery service is given by $80\sqrt[3]{x} + 500$, where x is the number of deliveries per day. In July the manager decides that it is necessary to keep delivery costs below Rs.1620. Find the greatest number of deliveries this company can make per day and still keep overhead below Rs .1620.

B) Of the equations $p + 3x = 39$ and $p = 9x + 9$, one is a supply function of a product and the other is a demand function of the product, where p is the price of the product and x is the quantity produced.

i) Sketch the two equations on the same axes.

ii) Label the demand and supply equations on the graph and give reasons for your choice.

iii) Find the equilibrium price and quantity.

C) Find an equation of the line through the point $(-5, 2)$ that is,

i) parallel to the line $2x + 5y = 7$

ii) perpendicular to the line $-3x + 4y = 5$

D) The circulation of a newspaper is increasing at a constant rate. Three months ago circulation was 3200. Today it is 4400.

i) Express the circulation in terms of time.

ii) What will be the circulation in 2months from today?

(20 Marks)

04. A) i) If $A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$ and $(A + B)^2 = A^2 + B^2$, find the value of a and b .

ii) A company manufactures five products and the company divided its sales into three sales districts. The matrix S below summarizes expected sales for each of the five products in each sales region for the coming month.

$$S = \begin{matrix} & \begin{matrix} \text{sales districts} \\ \hline \end{matrix} \\ \begin{matrix} \hline \\ \hline \\ \hline \\ \hline \\ \hline \end{matrix} & \begin{pmatrix} 500 & 200 & 350 \\ 400 & 300 & 100 \\ 250 & 425 & 50 \\ 100 & 150 & 350 \\ 200 & 175 & 225 \end{pmatrix} \end{matrix} \begin{matrix} \\ \\ \\ \\ \\ \end{matrix} \text{products}$$

The matrix R indicates the number of units of each component used in producing each product

$$R = \begin{matrix} & \begin{matrix} \text{Component} \\ \hline \end{matrix} \\ \begin{matrix} \hline \\ \hline \\ \hline \\ \hline \\ \hline \end{matrix} & \begin{pmatrix} 1 & 0 & 2 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 3 \\ 0 & 2 & 1 & 1 \\ 1 & 2 & 3 & 1 \end{pmatrix} \end{matrix} \begin{matrix} \\ \\ \\ \\ \end{matrix} \text{product}$$

The manufacture of each component requires the consumption of certain resources. The matrix P indicates the quantities of each of three standard parts and the number of production labour-hours and assembly labour hours used to produce one unit of each component.

Resource

<i>Part</i>	<i>part</i>	<i>part</i>	<i>prod.</i>	<i>asse.</i>
<i>I</i>	<i>II</i>	<i>III</i>	<i>hour</i>	<i>hour</i>

$$P = \begin{matrix} & \begin{matrix} \text{Resource} \\ \hline \end{matrix} \\ \begin{matrix} \hline \\ \hline \\ \hline \\ \hline \end{matrix} & \begin{pmatrix} 2 & 0 & 1 & 2 & 3 \\ 1 & 3 & 2 & 5 & 1 \\ 0 & 2 & 1 & 4 & 2 \\ 0 & 4 & 1 & 1 & 6 \end{pmatrix} \end{matrix} \begin{matrix} \\ \\ \\ \end{matrix} \text{component}$$

The matrix $C = (25 \ 15 \ 30 \ 10 \ 8)$ contains the cost (in Rs.) of the five resource part I, part II, part III, production labour, and assembly labour.

You are required to calculate the following using the appropriate matrix algebra:

- a. Multiplication S by the matrix $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and interpret it.
- b. The quantities need for each of the four components

- c. The resource requirements to produce the four components
- d. The total cost of producing the quantities of the five products needed for the month.

B) John has Rs. 25000 and invested part of it in fund A, part in fund B, and part in fund C. After one year, he received a total of Rs. 1620 in simple interest from the three investments. Fund A paid 6% annually, fund B paid 7% annually, and fund C paid 8% annually. There was Rs. 6000 more invested in fund B than fund C.

- i. Develop a system of three linear equations that can be used to find out the amount of money John invested in each category.
- ii. Solve the system of linear equations by matrix inversion method

(20 Marks)

05. A) i) Explain what is meant by the statement "The simple events that constitute a sample space are mutually exclusive and exhaustive".

ii) An investor has asked his stock broker to rate three stocks A, B and C and list them in the order in which he recommend them. Consider the following events:

L : Stock A doesn't receive the lowest rating
 M : Stock B doesn't receive the lowest rating
 N : Stock C receives the highest rating

- a. Define the random experiment and list the simple events in the sample space
 - b. List the simple events in each of the events L , M , and N
 - c. List the simple belonging to each of the following events: $(L \text{ or } M)$, $(L \text{ and } M)$ and \bar{N}
 - d. Identify a pair of exhaustive events among L , M , and N .
- iii) A store manager has cross - classified a sample of 250 customer purchases, as shown in the following table.

Size of purchase	Method of payment	
	Cash	Credit card
Under Rs. 20	51	31
Rs. 20 or More	65	103

- a) What is the probability that the customer selected paid by credit card?
- b) What is the probability that the customer selected made a purchase of under Rs.20?
- c) Are the events "payment by cash" and "purchase of under Rs.20" mutually exclusive? Explain.
- d) Are the events "payment by cash" and "purchase of under Rs.20" independent? Explain.

- B) i) The probability that a contractor will get a plumbing contract is $\frac{2}{3}$ and probability that he will not get an electric contract is $\frac{5}{9}$. If the probability of getting at least one contract is $\frac{4}{5}$, what is the probability that he will get both?
- ii) Suppose that a day's production schedule calls for 9000 items. Three machines A, B and C with a daily production capacity of 4000 have the probability that an item is defective on them as 1, 2 and 4 percent respectively. On a given day 4000 items were produced on A, 4000 on B and 1000 on C. one item is selected and found defective. What is the probability that it was produced on C?

(20 Marks)