

EASTERN UNIVERSITY, SRI LANKA

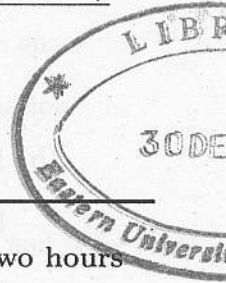
DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE -2008/2009

SECOND YEAR, SECOND SEMESTER (Jan./Apr., 2011)

EXTMT 218 - FIELD THEORY

(PROPER & REPEAT)



Answer all Questions

Time: Two hours

1. (a) A circular disc of radius  $a$  is charged uniformly with a charge density of  $\sigma$ . Find the electric field intensity at a point  $P$  at a distance  $h$  from the disc along its axis.
- Find the field at any point  $P$  at a distance  $h$  from the infinite plane sheet of charge  $\sigma$ .
  - Two infinite plane sheets are separated by a distance  $d$ . The first has a charge  $+\sigma$  and the second has a charge  $-\sigma$ . Find the electric field intensity at any point between them.
- (b) State the Gauss's law.

A spherical volume charge density distribution is given by

$$\rho = \begin{cases} \rho_0 \left(1 - \frac{r^2}{a^2}\right), & r \leq a; \\ 0, & r > a, \end{cases}$$

where  $a$  is the radius of the spherical volume.

- Calculate the total charge  $Q$ .
- Find the electric field intensity  $E$  outside the charge distribution.
- Find the electric field intensity  $E$  inside.

2. (a) Define an electric field strength due to a point charge.

Show that, the magnitude  $E$  of the electric field at a distance  $y$  along the perpendicular bisector of a thin non-conducting rod of finite length  $L$  with a charge  $q$  spread uniformly along it is given by

$$E = \frac{q}{2\pi\epsilon_0 y(L^2 + 4y^2)^{\frac{1}{2}}}$$

- (b) Define the term electric dipole.

Prove that the electric potential  $V$  at a point  $P$  at a distance  $r$  from the dipole of moment  $\underline{P}$  is given by

$$V = -\frac{1}{4\pi\epsilon_0} \left\{ \underline{P} \cdot \text{grad} \left( \frac{1}{r} \right) \right\}$$

and the electric field due to a dipole is given by

$$\underline{E} = \frac{1}{4\pi\epsilon_0} \left\{ \frac{3(\underline{P} \cdot \underline{r})\underline{r}}{r^5} - \frac{\underline{P}}{r^3} \right\}$$

3. Show by using separation of variables or otherwise, that the solution of the equation  $\nabla^2 V = 0$ , where  $V$  is the potential function in two dimensional rectangular coordinates is given by

$$V(x, y) = (A \sin(kx) + B \cos(kx))(C \exp(ky) + D \exp(-ky))$$

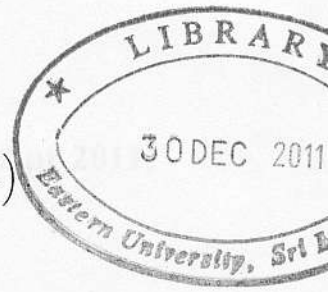
where  $A, B, C, D$  and  $k$  are arbitrary constants.

Prove that the potential distribution inside the rectangular region subject to the boundary conditions

- i.  $V = 0$ , when  $x = 0$ ,
- ii.  $V = 0$ , when  $x = a$ ,
- iii.  $V = V_0$ , when  $y = 0$ ,

iv.  $V \rightarrow 0$  as  $y \rightarrow \infty$ , is given by

$$V(x, y) = \frac{4V_0}{\pi} \sum_{n=1,3,5,\dots} \frac{1}{n} \exp\left(\frac{-n\pi y}{a}\right) \sin\left(\frac{n\pi x}{a}\right)$$



4. (a) Define the magnetic flux density  $\underline{B}$  and show that  $\text{div } \underline{B} = 0$  in space.

By assuming the Ampere's law in integral form deduce the equation

$$\text{Curl } \underline{B} = \mu_0 \underline{J}, \text{ where } \underline{J} \text{ is the current density.}$$

- (b) Define the magnetic field strength  $\underline{H}$  in a magnetizable media and show that

$$\text{Curl } \underline{H} = \underline{J}.$$

If no current are present and the magnetization is linearly proportional to  $\underline{H}$ , show that there exists a function  $\phi$  such that  $\nabla^2 \phi = 0$ .

- (c) A rod with mass  $m$  and a radius  $R$  is mounted on a two parallel rails of length  $a$  separated by a distance  $l$ . The rod carries a current  $I$  and rolls without slipping along the rails which are placed in a uniform magnetic field  $\underline{B}$  directed into the page. If the rod is initially at rest, show that the speed as it leaves the rails is

$$\sqrt{\frac{4IlBa}{3m}}.$$