EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE -2008/2009

SECOND YEAR, SECOND SEMESTER (Jan./Apr., 2011)

EXTMT 218 - FIELD THEORY

(PROPER & REPEAT)

LIBR
30DE
Eastern Universi

Answer all Questions | Time: Two hours

1. (a) A circular disc of radius a is charged uniformly with a charge density of σ. Find the electric field intensity at a point P at a distance h from the disc along its axis.

   i. Find the field at any point P at a distance h from the infinite plane sheet of charge σ.

   ii. Two infinite plane sheets are separated by a distance d. The first has a charge $+\sigma$ and the second has a charge $-\sigma$. Find the electric field intensity at any point between them.

   (b) State the Gauss's law.

   A spherical volume charge density distribution is given by

$$\rho = \begin{cases} \rho_0 \left(1 - \dfrac{r^2}{a^2}\right), & r \leq a; \\ 0, & r > a, \end{cases}$$

   where a is the radius of the spherical volume.

   i. Calculate the total charge Q.

   ii. Find the electric field intensity E outside the charge distribution.

   iii. Find the electric field intensity E inside.

2. (a) Define an electric field strength due to a point charge.

Show that, the magnitude E of the electric field at a distance y along the perpendicular bisector of a thin non-conducting rod of finite length L with a charge q spread uniformly along it is given by

$$E = \frac{q}{2\pi\epsilon_0 y(L^2 + 4y^2)^{\frac{1}{2}}}.$$

(b) Define the term electric dipole.

Prove that the electric potential V at a point P at a distance r form the dipole of moment $\underline{P}$ is given by

$$V = -\frac{1}{4\pi\varepsilon_0}\left\{\underline{\mathrm{P}} \cdot grad\left(\frac{1}{r}\right)\right\}$$

and the electric field due to a dipole is given by

$$\underline{\mathrm{E}} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{3(\underline{\mathrm{P}} \cdot \underline{\mathrm{r}})\underline{\mathrm{r}}}{r^5} - \frac{\underline{\mathrm{P}}}{r^3}\right\}.$$

3. Show by using separation of variables or otherwise, that the solution of the equation $\nabla^2 V = 0$, where V is the potential function in two dimensional rectangular coordinates is given by

$$V(x, y) = (A\sin(kx) + B\cos(kx))(C\exp(ky) + D\exp(-ky))$$

where A, B, C, D and k are arbitrary constants.

Prove that the potential distribution inside the rectangular region subject to the boundary conditions

i. $V = 0$, when $x = 0$,

ii. $V = 0$, when $x = a$,

iii. $V = V_0$, when $y = 0$,

LIBRARY
30 DEC 2011
Eastern University, Sri Lanka

iv. $V \to 0$ as $y \to \infty$, is given by

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5,\dots} \frac{1}{n} \exp\left(\frac{-n\pi y}{a}\right) \sin\left(\frac{n\pi x}{a}\right)$$

4. (a) Define the magnetic flux density $\underline{B}$ and show that $div\ \underline{B} = 0$ in space.

   By assuming the Ampere's law in integral form deduce the equation

   $Curl\ \underline{B} = \mu_0\ \underline{J}$, where $\underline{J}$ is the current density.

   (b) Define the magnetic field strength $\underline{H}$ in a magnetizable media and show that

   $Curl\ \underline{H} = \underline{J}$.

   If no current are present and the magnetization is linearly proportional to $\underline{H}$, show that there exists a function ϕ such that $\nabla^2\phi = 0$.

   (c) A rod with mass m and a radius R is mounted on a two parallel rails of length a separated by a distance l. The rod carries a current I and rolls without slipping along the rails which are placed in a uniform magnetic field $\underline{B}$ directed into the page. If the rod is initially at rest, show that the speed as it leaves the rails is

$$\sqrt{\frac{4IlBa}{3m}}.$$