2. (a) Prove the followings:

   i. the rate of change of total angular momentum $\underline{H}$ about a point O is equal t the total moment of the external forces about O,

   ii. $\underline{H} = \underline{r}_G \wedge M\underline{v}_G + \underline{H}_G$,

   where $\underline{H}_G$ is the angular momentum about the centre of mass and $\underline{v}_G$ is th velocity of the centre of mass.

(b) A uniform sphere of mass M and radius a is released from rest on a plane incline at an angle α to the horizontal. If the sphere rolls down without slipping, show tha the acceleration of the center of the sphere is constant and is equal to $\frac{5}{7}\, g\sin\alpha$.

3. (a) With the usual notations, state the Euler's equation for the motion of a rigid bod with one point fixed.

A body moves about a point O under no forces. The principal moments of inertia a O being 3A, 5A and 6A. Initially the angular velocity has components $\omega_1 = n$, $\omega_2 =$ 0, $\omega_3 = n$ about the corresponding principal axes. Show that at any time t,

$$\omega_2 = \frac{3n}{\sqrt{5}}.\tan(\frac{nt}{\sqrt{5}})$$

and that the body ultimately rotates about the mean axis.

4. (a) Define the Hamiltonian interms of the Lagrangian.

Hence show that the Hamiltonian's equations are given by

$$\dot{q}_j = \frac{\partial H}{\partial Pj}, \qquad \dot{P}_j = -\frac{\partial H}{\partial q_j},$$

when H does or does not contain the variable time t explicitly.

(b) If the Hamiltonian H is independent of time t explicitly, then prove that it is

   i. a constant,

   ii. equal to the total energy of the system.

(c) A particle moves in the xy plane under the influence of a central force depending only on its distance from the origin.

i. Set up the Hamiltonian for the system.

ii. Write the Hamiltonian's equations of motion.

5. (a) With the usual notations, derive Lagrange's equation for the impulsive motion from Lagrange's equation for a holonomic system in the following form

$$\Delta\left(\frac{\partial T}{\partial \dot{q}_j}\right) = S_j, \qquad j = 1, 2, \cdots, n.$$

(b) A square $ABCD$ formed by four equal rods, each of length $2l$ and mass m joined smoothly at their ends, rests on a smooth horizontal table. An impulse of magnitude I is applied to the vertex A in the direction of AD.

i. Find the equation of motion of the frame.

ii. Show that the kinetic energy of the square immediately after the application of impulse is $\dfrac{5I^2}{16m}$.

6. (a) Define the Poisson Bracket.

With the usual notations show that

$$\frac{dF}{dt} = [F, H] + \frac{\partial F}{\partial t}$$

for a function $F = F(p_j, q_j, t)$, $j = 1, 2, \cdots, n$. Prove the Poisson's theorem that $[F, G]$ is a constant of motion when $F = F(p_j, q_j, t)$ and $G = G(p_j, q_j, t)$, $j = 1, 2, \cdots, n$ are constant of motion.

(b) Find the frequency of oscillation of a particle of mass m which is moving along a line and is attached to spring whose other end is fixed at a point A at a distance l from the line.

EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE - 2008/2009

THIRD YEAR, SECOND SEMESTER (Jan./Apr., 2010)

EXTMT 307 - CLASSICAL MECHANICS III

Answer all Questions

Time: Three hours

1. A projectile located at colatitude λ is fixed with velocity v_0 in a southward direction at an angle α with the horizontal.

   (a) Find the position of the projectile after a time t.

   (b) Prove that after time t the projectile is deflected toward the east of the original vertical plane of motion by the amount

   $$\frac{1}{3}\omega g \sin\lambda t^3 - \omega v_0 \cos(\alpha - \lambda)t^2$$

   where ω is the rotation speed of the earth.

   (c) Prove that when the projectile returns to the horizontal, it will be at the distance

   $$\frac{4\omega v_0^3 \sin^2\alpha}{3g^2}\{3\cos\alpha\cos\lambda + \sin\alpha\sin\lambda\}.$$