EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
EXTERNAL DEGREE EXAMINATION IN SCIENCE - 2005/2006
SECOND YEAR FIRST SEMESTER(Mar./May, 2010)
EXTMT 215 - CLASSICAL MECHANICS II

Answer all Questions

Time: One hour

1. (a) Define the terms **common catenary** and the **parameter** of a catenary.

   (b) Suppose that an equation of a chain of variable weight is described by a part of a cycloid $x = a(t+\sin t)$ and $y = a(1-\cos t)$, where a is a constant. Find the intrinsic equation of the chain.

   (c) The chain mentioned in part (b) obeys the differential equation

   $$\frac{dT}{d\psi} + \mu T = -2a\omega[\sin 2\psi + \mu(1+\cos 2\psi)],$$

   where the notations used here are in the usual meaning, and satisfying the end conditions: $T(0) = P$ and $T(\pi/2) = 0$. Show that

   $$P = \frac{4a\omega}{4+\mu^2}\left(3e^{\mu\pi/2} - \mu^2 - 1\right),$$

   where e denotes the exponential function.

2. (a) Suppose that a beam AB is kept horizontally and carries continuously distributed load $\omega(x)$ per unit length. If M is bending moment at a breaking point P such that $AP = x$, show that

   $$\frac{d^2M}{dx^2} = -\omega(x).$$

   (b) Suppose $y = y(x)$ is an equation of a loaded beam AB such that the end A is clamped and B is kept as free. State what does it result for y_A, y'_A and M_B, where the notations given here are defined with the usual meaning.

   How do these change if the ends are simply supported?

(c) State the Clapeyron's equation.

A heavy uniform elastic beam AB of length $6a$ rests on five supports: two of th are at the ends A and B, one is at the middle point C and other two supports at D and E such that $AD = EB = a$, which are in the same horizontal line. S that the moments at C and D are respectively given by $aW/16$ and $aW/24$, wh W is the weight of the beam.