EASTERN UNIVERSITY, SRI LANKA

EXTERNAL DEGREE

THIRD EXAMINATION IN SCIENCE (Term System)-1999/2000

(MAY/JUNE,2010)

EXTMT- 306 - PROBABILITY AND STATISTICS II

(RE-REPEAT)

Answer five questions only — Time: Three hours

---

1. (a) Define the following terms:

   i. Sample space;

   ii. Mutually exclusive events.

   (b) Define the term "conditional probability".

   Hence show that

   If A and B are two mutually exclusive events and $(A \cup B) \neq 0$, show that

   $$P(A|(A \cup B)) = \frac{P(A)}{P(A)+P(B)}.$$

   (c) State and prove the Baye's Theorem.

   A bottle manufacturing company uses three machines A, B and C. Of their respective output 5%, 4% and 2% of the items are faulty. A manufactures 25%, B manufactures 35% and C manufactures 40% of the total output. A bottle drawn at random from the product line is found to be faulty. What is the probability that it was manufactured by B?

2. (a) Define the term "Moment generating function" of a random variable X.

Hence show that if X and Y are independent random variables, then X

has the Moment generating function,

$$M_{X+Y}(t) = M_X(t)\ M_Y(t)$$

where M_X and M_Y are moment generating function of X and Y, respecti

and t is a real variable.

(b) Let X be any continuous random variable with the probability density func

$f(x)$ given by

$$f(x) = \begin{cases} x + \frac{1}{2}, & 0 \leq x \leq 1; \\ 0, & \text{otherwise.} \end{cases}$$

Find the moment generating function of X.

(c) If X is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{x}{6} + \frac{1}{12}, & 0 \leq x \leq 3; \\ 0, & \text{otherwise,} \end{cases}$$

find the probability density function $g(y)$ and the cumulative distribution fu

tion $G(y)$ of $Y = 5X + 3$.

3. (a) If $X \sim Bin(n, p)$ then prove that

(i) $E(X) = np$,

(ii) $Var(X) = npq$.

(b) A pair of dice is thrown 10 times. If getting a doublet is considered a succ

find the probability of

(i) 4 successes,

(ii) No success.

(c) The difference between the mean and the variance of a Binomial distribut

is 1 and the difference between their squares is 11. Find n.

(d) A certain type of missile hits its target with probability $p = 0.3$. Find the number of missiles that should be fired so that there is atleast a 80 percent probability of hitting the target.

4. (a) Random variables X and Y have joint distribution function

$$f_{XY}(x,y) = \begin{cases} c(x^2 + \frac{1}{2}xy), & 0 < x < 1,\ 0 < y < 1; \\ 0, & \text{otherwise.} \end{cases}$$

Find

i. the value of c,

ii. the joint distribution function of X and Y,

iii. marginal density function of X.

(b) Let the random variables X and Y have the joint probability density function

$$f_{XY}(x,y) = \begin{cases} e^{-x-y}, & \text{if } x, y > 0; \\ 0, & \text{otherwise.} \end{cases}$$

and let $U = X + Y$ and $V = \frac{X}{X+Y}$.

i. Find the joint probability density function of U and V.

ii. Are U and V are independent random variables?

5. (a) i. Let A and B be two events. Show that

$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$.

ii. The National cricket team of a country has a constant probability 0.8 of winning a match played at their home country and 0.6 of winning a match played aboard. During this season, the team plays six matches in there home country and six matches abroad. Calculate the mean and variance of the number of matches that will be won by the team during the season.

(b) Show that the following statements are equivalent.

i. A and B^c are independent,

ii. A and B are independent,

iii. A^c and B^c are independent,

iv. $P(A \bigcup B) = 1 - P(A^c)P(B^c)$.

6. (a) Define the following terms of a random variable X.

i. rth moment about the origin,

ii. rth moment about the mean.

Hence show that $E[X^2] = Var(X) + (E[X])^2$.

(b) A telephone switch board receives on the average four calls per minute.

calls follows a poisson distribution

i. what is the probability exactly 8 calls per minute?

ii. what is the probability more than 10 calls per minute?

(c) A book contains 100 typing errors distributed randomly throughout

pages. Find the probability that

i. a page selected at randomly contains at least two errors,

ii. 10 pages selected at random contain 5 or more errors.

7. Define the term *unbiasedness.*

(a) Consider a distribution having a normal population with mean μ and va

σ^2.

i. Show that the sample mean $\overline{X}$ is an unbiased estimator for μ.

ii. Let $s_1^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$ and $s_2^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ be estimat

σ^2. Check the unbiasedness of s_1^2 and s_2^2.

(b) Define the term "*likelihood function*"

i. Let $X_1, X_2, \ldots\ldots X_n$ be n independent observations from normal distribution with unknown mean μ and unknown variance σ^2.

ii. Let us assume that the sample data follow a normal distribution with unknown mean μ and unknown variance σ^2.If n=12, $\sum_{i=1}^{12} X_i = 180$ and $\sum_{i=1}^{12} X_i^2 = 2799$. Find the maxzimum likelihood estimators for μ and σ^2.

8. (a) 11 students and 14 professors took part in a study to find mean commuting distances. The mean number of miles travelled by students was 5.6 and the standard deviation was 2.8. The mean number of miles travelled by professors was 14.3 and the standard deviation was 9.1. Assumed that the standard deviation are approximately equal and the two distributions are approximately normal. Construct a 95 percent confidence interval for the difference between the mean.

(b) Fifteen children are randomly selected from all 10 year old children in a town. Each child is then matched with another (previously unselected) child for age, sex, maturation and spelling ability. One child, in each of the 15 pairs of children is randomly allocated to be taught how to spell a list of 100 difficult words using method 1. The other child in each pair is taught by method 2. Following the learning period each child is given a spelling test. Estimate the mean difference in the scores for the two methods, assuming that these difference are normally distributed. Suppose the (sample) scores are as shown in table below.

Pair No	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Method 1 score	65	65	70	69	62	66	64	62	63	61	66	76	60	69
Method 2 score	61	66	66	63	59	66	57	57	60	59	65	69	58	65