EASTERN UNIVERSITY, SRI LANKA

EXTERNAL DEGREE

THIRD EXAMINATION IN SCIENCE (Term System)-1999/2000

(May/June, 2010)

EXTMT 307 - CLASSICAL MECHANICS III

RE-REPEAT

Answer five questions only. **Time: Three hours**

1. Two frames of reference S and S' have a common origin O and S' moves with angular velocity $\underline{\omega}$ relative to S. A moving particle P has a position vector $\underline{r}$ relative to O. Establish the formulas

$$\frac{d\underline{r}}{dt} = \frac{\partial \underline{r}}{\partial t} + \underline{\omega} \Lambda \underline{r},$$

$$\frac{d^2\underline{r}}{dt^2} = \frac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega}\Lambda\frac{\partial \underline{r}}{\partial t} + \frac{\partial \underline{\omega}}{\partial t}\Lambda \underline{r} + \underline{\omega}\Lambda(\underline{\omega}\Lambda\underline{r}),$$

by carefully defining the derivatives.

If a particle of mass m is thrown vertically upwards at a latitude λ with initial speed v_o, then prove that when it returns it will be at a distance

$$\frac{4\omega v_o^3 \cos\lambda}{3g^2}$$

from its starting point, where ω is the constant angular velocity of the earth with order of ω^2 negligible.

2. (a) Define the following terms:

   i. linear momentum;

   ii. angular momentum;

   iii. moment of force.

(b) Obtain the equation of motion with the usual notations for an N system of par

in the following form

$$\underline{\Gamma}_A = (\underline{r}_G - \underline{r}_A)\Lambda M \underline{f}_G + \frac{d\underline{H}_G}{dt},$$

where $\underline{\Gamma}_A = \sum_{i=1}^{N}(\underline{r}_i - \underline{r}_A)\Lambda \underline{F}_i$ is the moment of the external forces about a mo

point A in the system.

A solid of mass M is in the form of a tetrahedron $OXYZ$, the edges OX, OY and

of which are mutually perpendicular, rests with XOY on a fixed smooth horizo

plan and YOZ against a smooth vertical wall. The normal to the rough face X

is in the direction of a unit vector $\underline{n}$. A heave uniform sphere of mass m and cer

C rolls down the face causing the tetrahedron to acquire a velocity -$V_{\underline{j}}$, where

the unit vector along OY. If $\overrightarrow{OC} = \underline{r}$, then prove that

$$(M+m)V - m\dot{\underline{r}}\cdot\underline{j} = \text{constant}$$

and

$$\frac{7}{5}\ddot{\underline{r}} = \underline{f} - \underline{n}(\underline{n}\cdot\underline{f}),$$

where $\underline{f} = \underline{g} + \dot{V}\underline{j}$ and $\underline{g}$ is the acceleration of gravity.

3. With the usual notations, obtain Euler's equations of motion for a rigid body having a point fixed, in the following form:

$$A\dot{\omega}_1 - (B - C)\omega_2\omega_3 = M_1,$$

$$B\dot{\omega}_2 - (C - A)\omega_1\omega_3 = M_2,$$

$$C\dot{\omega}_3 - (A - B)\omega_1\omega_2 = M_3.$$

A rigid body possessing axial symmetry and with principal moments of inertia A, A, C at its center of mass is surrounded by a medium that produces a retarding couple which at any time is a constant multiple, k, of the instantaneous angular velocity. Show that if the principal axes at G are taken as axes of reference fixed in the body and are such that the initial angular velocity is $(\Omega_1, 0, \Omega_3)$,then the angular velocity at any subsequent time t is given by

$$\left(\Omega_1 e^{-\frac{k}{A}t}\cos\theta,\ \Omega_1 e^{-\frac{k}{A}t}\sin\theta,\ \ \Omega_3 e^{-\frac{k}{C}t}\right),$$

where $kA\theta = (C - A)C\Omega_3(1 - e^{-\frac{k}{C}t})$.

Hence show that the motion of spin about the axis of symmetry is stable or unstable according as $C \gtrless A$.

4. (a) With the usual notations, show that;

$$s + \dot{\phi}\cos\theta = \text{constant} = \text{n},$$

$$A\dot{\phi}\sin^2\theta + Cn\cos\theta = \text{constant} = \text{k},$$

$$A(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta) + 2mgh\cos\theta = \text{constant},$$

for the motion of a top with its tip on a perfectly rough horizontal floor, where s is the spin angular velocity of the top.

(b) If $u = \cos\theta$ then prove that:

i. $\dot{u}^2 = (\alpha - \beta u)(1 - u^2) - (\gamma - \delta u)^2 = f(u)(\text{say})$,

where $\alpha = \dfrac{2E - Cn^2}{A}$, $\beta = \dfrac{2mgh}{A}$, $\gamma = \dfrac{k}{A}$ and $\delta = \dfrac{\text{Cn}}{\text{A}}$;

ii. $t = \displaystyle\int \frac{du}{\sqrt{f(u)}} +$ constant.

(c) Find the condition for steady precession of a top without nutation and show two precessional frequencies are possible.

5. Obtain the Lagrange's equations of motion using D'Alembert's principle for a holono system.

A bead of mass M slides freely on a frictionless circular wire of radius b that rot in a horizontal plane about a point on the circular wire with a constant angular velo ω. Write down Lagrange's equation of motion of the bead, and show that the bead os lates as a pendulum of length $l = \dfrac{g}{\omega^2}$.

6. (a) With the usual notations, derive the Lagrange's equation for the impulsive mot from the Lagrange's equation for a holonomic system.

(b) A square $ABCD$ formed by four equal rods, each of length $2l$ and mass m joi smoothly at their ends, rests on a smooth horizontal table. An impulse of magnitu I is applied to the vertex A in the direction of AD.

i. Find the equation of motion of the frame.

ii. Show that the kinetic energy of the square immediately after the application impulse is $\dfrac{5I^2}{16m}$.

7. (a) Define the Poisson Bracket.

With the usual notations show that

$$\frac{dF}{dt} = [F, H] + \frac{\partial F}{\partial t}$$

for a function $F = F(p_j, q_j, t)$, $j = 1, 2, \cdots, n$. Prove the Poisson's theorem that $[F, G]$ is a constant of motion when $F = F(p_j, q_j, t)$ and $G = G(p_j, q_j, t)$, $j = 1, 2, \cdots, n$ are constant of motion.

(b) For a certain system with two degree of freedom, the hamiltonian is given by

$$H = \eta^2(p_1^2 + p_2^2) + \nu^2(p_1 q_1 + p_2 q_2)^2$$

where η and ν are constants.

Show that if H is a constant and $F = p_1 q_1 + p_2 q_2$ then

$$[F, H] = 2(H - \nu^2 F^2).$$

. (a) Define the Hamiltonian interms of the Lagrangian.

Hence show that the Hamiltonian's equations are given by

$$\dot{q}_j = \frac{\partial H}{\partial Pj}, \qquad \dot{P}_j = -\frac{\partial H}{\partial q_j},$$

when H does or does not contain the variable time t explicitly.

(b) If the Hamiltonian H is independent of time t explicitly, then prove that it is

i. a constant.

ii. equal to the total energy of the system.

(c) A particle moves in the xy plane under the influence of a central force depending only on its distance from the origin.

i. Set up the Hamiltonian for the system.

ii. Write the Hamiltonian's equations of motion.