EASTERN UNIVERSITY, SRI LANKA

FIRST EXAMINATION IN SCIENCE 2004/2005

FIRST SEMESTER(May/June' 2008)

EXTERNAL DEGREE

EXTMT 101 - FOUNDATION OF MATHEMATICS

(Proper and Repeat)

Answer all questions **Time: Three hours**

Q1. (a) Let p and q be any two statements. Prove each of the following logical identities by using the laws of algebra of propositions.

i. $p \vee (p \wedge q) \equiv p$;

ii. $> (p \vee q) \vee (> p \wedge q) \equiv \ > p$.

(b) Test the validity of the following argument.

If I study, then I will not fail the exam.

If I do not play games, then I will study.

But I failed the exam.

....

Therefore I must have played games.

Q2. (a) Let the operator Δ be defined by $A \Delta B = (A \setminus B) \cup (B \setminus A)$.

Show that

i. $A \Delta B = (A \cup B) \setminus (A \cap B)$;

ii. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.

(b) Prove that $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$.

Is it true that $A \cup (B \setminus C) = (A \cup B) \setminus (A \cup C)$? Justify your answer.

(c) 60 students sat an examination offering Mathematics, Physics, chemistry. 40 students passed Mathematics, 39 students passed physics and 38 students passed chemistry. Given that the number of students who passed one subject is 6. Show that at most 37 students passed all three subjects and at least 23 students failed at least one subject.

Q3. (a) What is meant by an equivalence relation?

A relation R is defined on $\mathbb{N}$, the set of all natural numbers, by

$$xRy \Leftrightarrow \exists\, n \in \mathbb{Z} \text{ such that } x = 2^n y, \text{ where } \mathbb{Z} \text{ set of all integers.}$$

Prove that R is an equivalence relation.

(b) Let A be a set and let R be an equivalence relation on A. Prove that

i. $[a] \neq \phi, \forall\, a \in A$;

ii. $aRb \Leftrightarrow [a] = [b], \forall\, a, b \in A$;

iii. $b \in [a] \Leftrightarrow [a] = [b], \forall\, a, b \in A$;

iv. either $[a] = [b]$ or $[a] \cap [b] = \phi, \;\forall\, a, b \in A$.

Q4. (a) Define the following terms

i. injective mapping;

ii. surjective mapping;

iii. bijective mapping.

(b) Prove that, if $f : S \to T$ is an injective mapping then $f(A \cap B) = f(A) \cap f(B), \;\; \forall\, A, B \subseteq S$.

(c) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} x^4 & \text{if } x \geq 0, \\ -x(x-2) & \text{if } x < 0. \end{cases}$

Prove that f is a bijective mapping and find it's inverse.

Q5. (a) Define the following terms

i. partially ordered set;

ii. totally ordered set;

iii. first element of a partially ordered set;

iv. maximal element of a totally ordered set.

(b) i. Show that every partially ordered set has at most one first element and at most one last element.

ii. Let $A = \{2, 3, 4, 6, 8, 16, 32, 64\}$ and a relation R on A be defined by $xRy \Leftrightarrow x$ divideds y. Find the supremum and infimum (if exists) for a subset $B = \{2, 4, 8\}$ of A.

Q6. (a) Define the following

i. group;

ii. abelian group.

(b) For any group G, prove that the following conditions are equivalent.

i. G is abelian;

ii. $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$;

iii. $(ab)^2 = a^2b^2$ for all $a, b \in G$.

(c) Define the following

i. The greatest common divisor(gcd) of two integers a and b ;

ii. The least common multiple(lcm) of two integers a and b.

Prove that $lcm(a, b) = \dfrac{|ab|}{gcd(a, b)}$, where a and b are non-zero integers.