



EASTERN UNIVERSITY, SRI LANKA
EXTERNAL DEGREE FIRST EXAMINATION
IN SCIENCE 2003/2004
October, 2007
SECOND SEMESTER
EXTMT 105 - THEORY OF SERIES
Proper & Repeat

Answer all questions

Time: One hour

- Q1. (a) What is meant by the infinite series $\sum_{n=1}^{\infty} a_n$ is convergent.
- (b) State and prove the **Comparison Test** and **Limit form of Comparison Test**.
- (c) State and prove the **D'Alembert's Test**.
- (d) State the theorem of **Integral Test**.
- (e) Use the above tests and the **Alternating Series Test**, to determine whether the following series converge or diverge:

i. $\sum_{n=1}^{\infty} \sin\left(\frac{(n^2 + 1)\pi}{n}\right);$

ii. $\sum_{n=2}^{\infty} \frac{1}{n(\lg n)^3};$

iii. $\sum_{n=1}^{\infty} \frac{3^n (n!)^3}{(3n)!};$

iv. $\sum_{n=1}^{\infty} \frac{n^2 + 2}{3n^3 + 4n}.$

- Q2. (a) Let $f : A \subseteq \mathbb{R} \rightarrow \mathbb{R}$. Define what is meant by f uniformly convergent on A .

(b) The series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$ is a power series about 3. Find the interval of convergence and radius of convergence.

(c) Let $f_n : A \subseteq \mathbb{R}$ and let $f_n \rightarrow f$ uniformly on A and each f_n is continuous on A , then prove that f is continuous on A .

(d) Show that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \forall x$

(e) Find a power series about 0 that represents $\frac{x}{1-x^3}$.