



EASTERN UNIVERSITY, SRI LANKA
EXTERNAL DEGREE SECOND EXAMINATION IN
SCIENCE - 2002/2003
SECOND SEMESTER (October/November, 2007)
EXTMT 218 - FIELD THEORY

Answer all Questions

Time: Two hours

Q1. (a) With the usual notations, prove that

$$\vec{E} = -\vec{\nabla}\phi.$$

Hence, show that

$$\phi = \frac{Q}{4\pi\epsilon_0 r}.$$

[40 marks]

(b) A potential distribution is given by the expression

$$\phi = \frac{20}{(x^2 + y^2 + z^2)}.$$

Determine the electric field intensity \vec{E} in the general form and also the particular value at the point (5,3,0). [40 marks]

(c) What is meant by the following mathematical interpretation? Explain it.

$$\oint \vec{E} \cdot d\vec{r} = 0. \quad [20 \text{ marks}]$$

Q2. (a) State **Gauss law** of the electric field and write down its integral form for a continuous charge density. [20 marks]

(b) Obtain **Poission's equation** using part (a) and hence, find the relation of potential if it is a function of r , distance along the radial direction, only.

[45 marks]

(c) A uniform volume charge distribution of -10^{-8} coulomb/m³ occupies the region between two co-axial conducting cylinders of radii 20 and 50 mm. If the electric field and potential are both zero on the inner cylinder, find the potential on the outer cylinder. [Use the result obtained in part (b)]

[35 marks]

Q3. (a) Write down the integral and differential forms of **Ampere's law** of magnetic field.

[20 marks]

(b) Using Ampere's Law, prove that the following result:

$$(i) \nabla \times \vec{H} = \vec{J};$$

$$(ii) \oint_c \vec{H} \cdot d\vec{s} = I;$$

where \vec{H} and \vec{J} are magnetic field strength and current density, respectively.

[50 marks]

(c) Show that the magnetic field B due to an infinitely long conductor carrying steady current i through it, is,

$$B = \frac{\mu_0 i}{2\pi a},$$

where a is the radius of the loop.

[30 marks]

Q4. (a) Write down the Kepler's law of planetary motion.

[30 marks]

(b) Consider a particle of small mass m moves around another particle of large mass M . The mass m is attracted by M and M to be at rest. If (r, θ) is the polar coordinate of m with respect to M and G is the gravitational constant, prove that

$$\frac{d^2 u}{d\theta^2} + u = \frac{GM}{l^2},$$

where $u = \frac{1}{r}$ and l is a constant. If the general solution of the differential equation above is

$$u = c \cos(\theta + \theta_0) + \frac{GM}{l^2},$$

where c and θ_0 are arbitrary constants, prove that

$$s = \frac{l^2}{GM} \quad \text{and} \quad e = \frac{cl^2}{GM},$$

where s and e are the semi-latus rectum and eccentricity of the conic shape

$$r = \frac{s}{(1 + e \cos \theta)},$$

respectively. What can you say about the path of the mass m ? [70 marks]