EASTERN UNIVERSITY, SRI LANKA

EXTERNAL DEGREE EXAMINATION IN SCIENCE

FIRST YEAR FIRST SEMESTER -2003/2004

(Oct./ Nov., 2006)

MT 106 - TENSOR CALCULUS

Proper & Repeat

Answer all questions   Time : One hour

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1. (a) Write the transformation equation for the following tensors:

    i. A^{ms}_{qr},

    ii. B^{pqr}_{mn},

    iii. C_{ijk}.

   (b) Let A^{pq}_{rst} be a tensor.

    i. Choose $p = t$ and show that A^{pq}_{rsp}, where the summation convention is employed, is a tensor. What is it rank?

    ii. Choose $p = t$ and $q = s$ and show similarly that A^{pq}_{rqp} is a tensor. What is its rank?

   (c) Find g and g^{jk} corresponding to the line element
   $$ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3.$$

2. (a) Define the following:

    i. Christoffel symbols of the first and second kind;

    ii. Geodesic.

(b) Determine the Christoffel symbols of the second kind for the line element

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta \, d\phi^2,$$

and find the corresponding geodesic equations.

(c) With the usual notations, prove the following:

    i. $\dfrac{\partial g_{rs}}{\partial r^m} = [rm, s] + [sm, r]$,

    ii. $\dfrac{\partial g^{rs}}{\partial r^m} = -g^{rn}\Gamma^s_{mn} - g^{sn}\Gamma^p_{mn}$.

    Hence deduce that the covariant derivative of metric tensors g^{rk} and g_{rk} are zero.