EASTERN UNIVERSITY, SRI LANKA

EXTERNAL DEGREE EXAMINATION IN SCIENCE

SECOND YEAR FIRST SEMESTER - 2002/2003

(Oct./ Nov., 2006)

MT 207 - NUMERICAL ANALYSIS

Answer all questions | Time : Two hours

1. Define

   * absolute error;
   * relative error.

   Illustrate, with an example, the loss of significance phenomenon.

   (a) Suppose x whose actual value is 2.0 is measured as 2.05.

      i. Give the relative error occurred in measuring x.

      ii. Compute x^2, x^3, x^4 and find the relative error in each computation.

   (b) Let $Z = \sigma \times (0.a_1a_2...a_na_{n+1}...)_\beta \times \beta^e$, $\sigma = \pm 1$, $a_1 \neq 0$ be a number in the base β. Express the rounded machine version $fl(z)$ of z, where the number z is rounded to n digits.

      Show that

      $$\frac{|z - fl(z)|}{z} \leq \frac{1}{2}\beta^{1-n}$$

      Hence deduce that, $fl(z) = (1+\epsilon)z$ with $|\epsilon| \leq \frac{1}{2}\beta^{1-n}$.

2. Define the order of convergence of an iterative method to compute the roots of a non- linear equation

$$f(x) = 0.$$

(a) Obtain Newton-Raphson algorithm to compute the root of the equation $f(x) = 0$ in an interval $[a, b]$. Show that the order of convergence of Newton-Raphson algorithm is at least 2.

(b) Obtain Secant method to compute the root of the equation $f(x) = 0$ in an interval $[a, b]$. Show that the order of convergence of Secant method is approximately 1.62.

3. (a) Let $f : [a, b] \to \mathbb{R}$ and let $x_0, x_1, ..., x_n$ be distinct and lie in $[a, b]$. Define the Lagrange polynomials $l_0, l_1, ..., l_n$ for these interpolation points.
Prove that there exists a unique polynomial p of degree at most n, the Lagrange interpolation polynomial, such that

$$p(x_i) = f(x_i), \ i = 0, 1, ..., n.$$

(b) Let $f \in C^{n+1}[a, b]$ and p be the polynomial of degree n which interpolates f at the distinct points $x_0, x_1, ..., x_n$ in $[a, b]$. Let
$l(x) = (x - x_0)(x - x_1)...(x - x_n)$. Then show that for each $x \in [a, b]$, there exists $\xi \in (a, b)$ such that

$$f(x) - p_n(x) = \frac{l(x) f^{n+1}(\xi)}{(n + 1)!}.$$

4. Suppose you are required to compute

$$I = \int_a^b f(x)dx.$$

(a) Describe the **Trapezoidal method** to compute the value of I and derive a formula for the error. State the conditions that f should satisfy in order to apply Trapezoidal rule.

(b) With the usual notations, the **Simpson's rule** is given by

$$\int_{x_{i-1}}^{x_{i+1}} f(x)dx = \frac{h}{3}(f_{i-1} + 4f_i + f_{i+1}) - \frac{1}{90}h^5 f^{(iv)}(\eta_i), \ \eta_i \in [x_{i-1}, x_{i+1}].$$

Obtain the composite Simpson's rule and show that the composite error is

$$\frac{1}{180}h^4(b-a)f^{(iv)}(\xi), \ \xi \in [a, b].$$

(c) Describe **Gauss Elimination** with scaled partial pivoting. Use the following to illustrate your answer.

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$$