EASTERN UNIVERSITY, SRI LANKA

EXTERNAL DEGREE EXAMINATION IN SCIENCE

FIRST YEAR FIRST SEMESTER - 2003 / 2004

(Oct./Dec., 2006)

EXTMT 101 - FOUNDATION OF MATHEMATICS

(Proper & Repeat)

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swer all questions — Time allowed: Three hours

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1. (a) Let p, q and r be three statements. Prove each of the following by using the laws of algebra of propositions:

   i. $p \vee (q \wedge p) \equiv p$,

   ii. $[p \wedge (q \vee r)] \wedge \neg[(\neg q \vee \neg r) \wedge r] \equiv p \wedge q$.

   (b) Test the validity of the argument: "You can go out if and only if you do washing up. If you go out then you won't watch television. Therefore you either watch television or wash up but not both."

2. (a) For any sets A and B, prove that $A \triangle B = (A \cup B) \setminus (A \cap B)$.

   Hence show that:

   i. $(A \triangle B)$ and $(A \cap B)$ are disjoint,

   ii. $(A \triangle B) \cup (A \cap B) = A \cup B$.

   (b) Let $f : A \longrightarrow B$ be a function. Prove that the relation R_f defined on A by $xR_f y \Leftrightarrow f(x) = f(y)$ is an equivalence relation.

   Let $f : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R} \times \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}\right) & \text{if } (x,y) \neq (0,0) \\ (0,0) & \text{if } (x,y) = (0,0) \end{cases}$$

   Find the R_f class of $(1, 0)$.

3. Define each of the following:

   (a) an injective mapping,

   (b) a surjective mapping,

   (c) a composition of two mappings.

   The functions $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$ are defined by

$$f(x) = \begin{cases} 4x+1 & \text{if } x \geq 0, \\ x & \text{if } x < 0; \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 3x & \text{if } x \geq 0, \\ x+3 & \text{if } x < 0. \end{cases}$$

   Find $g \circ f$. Show that $g \circ f$ is a bijective mapping and find its inverse.

4. Prove that if $f : X \longrightarrow Y$ is a function and A and B are subsets of X, then $f(A \cup B) = f(A) \cup f(B)$ and $f(A \cap B) \subseteq f(A) \cap f(B)$.

   Prove also that

   $f(A \cap B) = f(A) \cap f(B)$ for all subsets A, B of X if and only if f is injective.

5. (a) Define each of the following terms:

      i. Partially ordered set,

      ii. Totally ordered set,

      iii. Last element,

      iv. Maximal element.

   (b) Show that every partially ordered set has at most one last element.

   (c) Show that last element of every partially ordered set is a maximal element. Is the converse true? Justify your answer.

   (d) Prove that in a totally ordered set every maximal element is a last element.

6. (a) Prove the following:

      i. If a is an arbitrary integer then $3 \mid a(a+1)(a+2)$,

      ii. If a is an odd integer then $8 \mid a^2 - 1$,

      iii. If a and b are odd integers then $8 \mid a^2 - b^2$.

   (b) A certain number of sixes and nines are added to give a sum of 126; if the numb of sixes and nines are interchanged, the new sum is 114. How many of each we originally?