EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 1996/97

(June/July' 2004)

EXTERNAL DEGREE

EXMT 205 & 208 - MATHEMATICAL METHODS & NUMERICAL ANALYSIS

Answer four questions only selecting two questions from each section

Time : Two hours

Section A

1. (a) Write the transformation equation for the following tensors.

   i. A_{jkl}

   ii. B^{ij}_{klm}

   iii. C^i_j

   (b) If A^{pq}_r and B^s_t are tensors, prove that $C^{pqs}_{rt} = A^{pq}_r B^s_t$ is also a tensor.

   (c) The covariant components of a tensor in rectangular co-ordinate system are $yz,\ 3,\ 2x+y$. Find its covariant components in spherical co-ordinates (r, θ, ϕ).

2. (a) Explain the terms "Covariant derivative" and "Absolute derivative" as applied to a tensor of type A^i_{jk}.

(b) Prove that the absolute derivative of δ^j_k, g_{jk}, g^{jk} are zero.

(c) Show that if A^{pq}_{rs} is a tensor, then $A^{pq}_{rs} + A^{qp}_{sr}$ is a symmetric tensor and $A^{pq}_{rs} - A^{qp}_{sr}$ is a skew symmetric tensor.

(d) Write the covariant derivative of $A^{jk}_i B^l_m$ with respect to x^q.

3. (a) Explain what is meant by the following terms.

i. Christoffel symbols of first and second kinds,

ii. Geodesics.

(b) With the usual notation, prove the following:

i. $[pq, r] = [qp, r]$,

ii. $\dfrac{\partial g_{pq}}{\partial x^m} = [pm, q] + [qm, p]$,

iii. $\dfrac{\partial g^{pq}}{\partial x^m} = -g^{pn}\Gamma^q_{mn} - g^{qn}\Gamma^p_{mn}$.

(c) Find the Christoffel symbols of the second kind for the line element

$$ds^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2$$

and find the corresponding Geodisic equations.

Section B

4. (a) Let $P_n(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n$, $a_0 \neq 0$ and let the sequence $b_0, b_1, \cdots, b_n$ be defined by

$$b_0 = a_0$$
$$b_i = tb_{i-1} + a_i, \quad i = 1, 2, 3, \cdots, n.$$

Show that the polynomial

$$P_{n-1}(x) = b_0x^{n-1} + b_1x^{n-2} + \cdots + b_{n-1}$$

is the quotient polynomial and the constant $P_n(t)$ is the remainder when $P_n(x)$ is divided by $(x - t)$.

Hence find the following:

i. the quotient polynomial and remainder when $P_4(x) = x^4 + 3x^3 + 4x + 1$ is divided by $(x - 2)$.

ii. the Taylor series of $P_4(x)$ about the point $x = 2$.

(b) Explain what is meant by

i. floating point representation,

ii. fixed point representation.

Round the following numbers in $FI(10, 3)$.

i. 4.13768, ii. -1.485.

Round the following numbers in $FL(10, 2)$.

i. 3.15, ii. -3.15.

5. Define the order of convergence of an iterative method to compute the roots of a non-linear equation

$$f(x) = 0 \cdots\cdots\cdots\cdots(1).$$

(a) Obtain Newton-Raphson algorithm to compute the roots of the equation (1) in an interval $[a, b]$.

Show that the order of convergence of Newton-Raphson algorithm is at least 2.

(b) Obtain Secant method to compute the roots of the equation (1) in an interval $[a, b]$.

Find $\sqrt{78.8}$ correct to 4 decimal places by using the methods (a) and (b).

6. (a) Let $f : [a, b] \to \mathbb{R}$ and let $x_0, x_1, \cdots, x_n$ be distinct points in $[a, b]$. Prove that there exists a unique polynomial P of degree at most n, the Lagrage interpolation polynomial, such that

$$p(x_i) = f(x_i), \quad i = 0, 1, 2, \cdots, n.$$

Find the Lagrange interpolation polynomial for $f(x) = \frac{1}{x}$ using the distinct points $x_0 = 2,\ x_1 = 2.5, x_2 = 4$.

(b) Let $f \in C^{n+1}[a, b]$ and p be the polynomial of degree n which interpolates f at the distinct points $x_0, x_1, \cdots, x_n$ in $[a, b]$.

Let $l(x) = (x - x_0)(x - x_1)\cdots(x - x_n)$. Then show that for each $x \in [a, b]$, there exists $c \in [a, b]$ such that

$$f(x) - p(x) = \frac{l(x)}{(n+1)!}\, f^{(n+1)}(c).$$

Let $p \in P$ interpolate at x_0 and $x_1 = x_0 + h$ and $f \in C^2[x_0, x_1]$ for $x_0 \leq x \leq x_1$ then show that $|f(x) - p(x)| \leq \frac{h^2}{8} \max |f^2(c)|$, $x_0 \leq c \leq x_1$.