EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST YEAR FIRST SEMESTER(2002/2003)

EXTERNAL DEGREE

MT 106 - TENSOR CALCULAS

Answer all questions   Time: One hour

1. (a) Explain what is meant by the following terms;
      i. Covariant tensor,
      ii. Contravariant tensor.

   (b) Write down the law of transformation for the following tensors:

      $(i)\ A^{ms}_{qr},$ $(ii)\ B^{ijk}_{lm},$ $(iii)\ C_{mn}.$

   (c) If $ds^2 = g_{jk}dx^j dx^k$ is an invariant, show that g_{jk} is a symmetric covariant tensor of rank two.

   (d) A covariant tensor has components xy, $2x - z^2$, xz in rectangular coordinates. Find its covariant components in spherical coordinates.

1

2. (a) Prove, with usual notation, that:

   i. $\dfrac{\partial g_{pq}}{\partial x^m} = [pm, q] + [qm, p]$,

   ii. $\dfrac{\partial g^{nk}}{\partial x^m} = -g^{in}\Gamma^k_{im} - g^{ik}\Gamma^n_{im}$,

   iii. $\Gamma^p_{pq} = \dfrac{\partial \ln \sqrt{g}}{\partial x^q}$.

(b) Explain the terms covariant derivative and absolute derivative as applied to a tensor of type A^i_{jk}.

   Calculate the absolute derivative of the following:

   i. A^{lmn}_{jk},

   ii. An invariant ϕ,

   iii. Metric tensor g_{ij}.